Evaluation of queuing Network Model Using Mean Value Algorithm

Pradeep K Joshi[1], Nidhi Sharma[2], R.K. Sharma[3], Pragya Shukla[2]

[1] Department of Mathematics, IPS Academy, Indore, India

[2] Research Scholar, Institute of Technology,, Devi Ahilya Vishwavidyalaya Indore, India

[3] Government Holkar Science College, Indore, India

Abstract:

The phenomenon of the network naturally exists naturally in different structure as post offices, bank, supermarket etc. In this paper, we provide a detailed theoretical analysis of network queues and applications in these networks structure. Out investigation is focused upon the theoretical researches state of art. We provide a description of the network queues resulting from an activity in the real world. On cyclic network queues some interesting directions of research and their potential solutions are also discussed. Within this paper the mean value algorithm is applied to evaluate the result variance, if there are 3 ticket counters vailable within particular railway station and the passengers arriving in the same ratio and the service is lacking then we measure the mean travel time in queue; where the mean occupancy seen by a customer arriving at queue i, finally calculate the absolute values of customer streams λ_i through different levels.

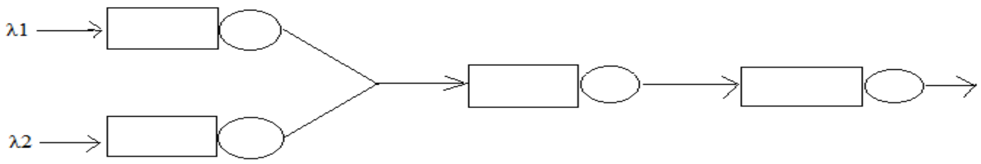
Keywords: Network queues, Cyclic Queue, Markovian Queue, Mean Value Algorithm (MVA).

Introduction:

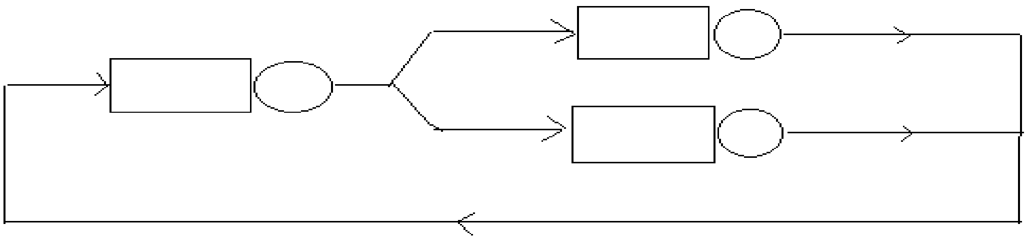
Because we know that the implementation of the queueing principle requires multiple queuein g approaches linked to each other as departures from one process turn out to be arrivals to ano ther process[1][3].Customer routing is the number of queues connected in the Networks of queues scheme. If a customer is serviced at a single node and can join every other node and queue for service, or exit the network[2]. In this paper, we provide an introduction to queueing networks with their methodologies of classifications, output indicators and some useful approximations for our analyses. All the queues interconnected with the jobs that flow from queue to queue. Example: machinery store, networks of contact, compute r system[4][8][10].The research on the queueing method and queueing networks begins from the XX century. There are n number of papers and the researchers have deals with the theory of queueing. It's just unlikely for all of them to provide full detail; so mean though we only mention the most important one once in our opinion. In 1909, a Danish engineer, Erlang, published his first paper on queueing theory, in the history of queueing theory. After that, in 1951, the term "queueing system" in D.G's article references. Kendall Kendall, In 1951, in D.G. Post.Kendall published then his paper on the queueing notation and D.R. in 1953. Cox published in 1955 on the study of the cycle of non-Markov. Then we get the new stuff over the queueing principle one by one. Jackson had considered in such a series exponential servers with open queueing networks in 1957 and a Poisson cycle. He was demonstrating that a product process is in the steady-state distribution. A sort of parallel queues that were implemented by F. 1958 Haight. In program from average response time with reliance on average number of queues for that J task. In 1961 a theory proved nothing, by W.J. Gordon and G.F. Newell. In the same year model of C.E. Skinner occurred. In 1968 M. Mandelbaum and B. Avi- Itzhak the concept of Fork-Join systems were introduced. By Buzen J.P. proposed the convolution algorithm to computation of the normalization constant, in 1973. A special case is presented network in which various types of jobs belong and have F.P's exponential distribution of service-time. Kim: Kim. There is a Poisson arrival cycle through the network on a fixed route and there each form has. J.P. Courtois, the decompositional approach was introduced in 1977. A large Jackson network in 1978- the form of queueing networks named by M.I. Reiman: Reiman. Exponentially distributed are not needed for the arrival and service times of network work intervals. S. Lavenberg, M. In 1980, Reiser created the Mean Value Analysis set of rules for evaluation of closed queueing networks. Network Representative founded in 1986 by S. Fdida Fedida. G-networks principles and E's incorporated positive and negative customers. 1991: Gelenbe. The development in the use of queueing theory for software modelling is found in recent years. Queueing network in applications are: Chemotherapy unit model; information system performance evaluation; human performance modeling; approximation of queueing networks use in diffusion.

Classification of queueing system: many communication systems must be modelled as a set of interconnected queues – which is termed a queueing network. Systems modelled by queueing networks can be grouped into such categories:

1) Open network- jobs arrive from external sources, circulate, and eventually depart. Example packet switched data network.



2) Closed network- multiple resources are sheared because of that there is no job enters or depart in such kind of model. It can also approximate a system involving multiple resources holding under heavy load fixed population of k jobs circulate continuously and never leave. Previous machine- repairman problem. Example CPU job scheduling problem



3) Mixed network- any combination of previous types. Example- for the virtual circuit simple model that is called window flow controlled.
4) Jackson network- on queueing networks the basic work did by James Jackson (UCLA Math professor). It is a special magnificence of open queueing networks – community of M queues, there is simplest one elegance of clients inside the community, a job can go away the community from any node, with price μi all service instances are exponentially allotted at queue i, for all nodes the service discipline is FCFS, all external customer arrival procedures with y_i at queue i.

Property 1. Let the routing matrix **P** describe the flow of jobs within a Jackson network, and let γ_i denote the mean arrival rate of jobs going directly into Node i from outside the network with γ being the vector of these rates. Then $\lambda = \gamma\,(\mathbf{I}-\mathbf{P})^{-1}$ where the components of the vector λ give the arrival rates into the various nodes; i.e., λ_i is the net rate into Node i.

Property 2. Consider a Jackson network containing m nodes with Node i having c_i (exponential) servers. Let Ni denote a random variable indicating the number of jobs at Node i (the number in the queue plus the number in the server(s)). Then, $\Pr\{N1 = n1, \cdots, Nm = nm\} = \Pr\{N1 = n1\}\times\cdots\times\Pr\{Nm = nm\}$, and the probabilities $\Pr\{N_i = n_i\}$ for $n_i = 0,1, \cdots$ can be calculated using the appropriate M/M/ci formula.

Property 3. Consider a Jackson network containing m nodes. Denote by N_{net} the total number of jobs within the network, T_{net} the time that a job spends in the network, and γ the vector of external arrival rates. Thus we have that

$$E[N_{net}] = \sum_{i=1}^{m} L_i, \text{ and}$$

$$E[T_{net}] = \frac{E[N_{net}]}{\sum_{i=1}^{m} \gamma(i)},$$

Where the individual L_i terms are given by the appropriate M/M/c formula.

Property 4. Let **P** denote the routing matrix associated with a closed queueing network containing m nodes. Form the matrix **Q** by setting it equal to **P** after deleting the first row and first column; that is, $q_{i,j} = p_{i+1,j+1}$ for i, j = 1, · · · , m−1. Then the vector of relative arrival rates, **r**, is given by $(r_2, \cdots, r_m) = (p_{1,2}, \cdots, p_{1,m})\,(\mathbf{I}-\mathbf{Q})^{-1}$ and $r_1 = 1$.

Jackson's Theorem for Open Queuing Networks

This is a special type of open queueing networks – M queue network, there is only one customer type in the network, a job can exit the network from any node, with rate μ_i all service times are spread exponentially in queue i, for all nodes the service discipline is FCFS single server queues, Poisson at rate $r_1, r_2, \ldots, r_K$, service customer times in j^{th} queue are exponentially at average $1/\mu_j$ and are mutually independent and separate for the arrival processes, when a customer is served in queue i, he enters each queue j with probability Pij or leaves the system with probability of $1 - \sum_{j=1}^{k} P_{ij}$. The routing probability from node i to node j is called P_{ij}. For all possibilities of i and j.

The following equations are:

$$\lambda_j = r_j + \sum_{i=1}^{k} \lambda_i P_{ij}, \qquad j = 1, \ldots, k$$

Solve the linear equations that have been obtained, and λ_i are. Defining the utilization factor for every queue as

$$\rho_j = \frac{\lambda_j}{\mu_j}, \qquad j = 1, \ldots, k$$

Then we have:

Jackson's Theorem. Assuming that $\rho_j < 1, j = 1, \ldots, K$, we have for all $N_1, \ldots, N_K \geq 0$,

$$P(N_1, \ldots, N_k) = P_1(N_1)\, P_2(N_2) \ldots P_k(N_k)$$

where

ISSN: 2278-4632
Vol-10 Issue-6 No. 13 June 2020

$$P_j(N_j) = \rho_j^{N_j}(1-\rho_j), N_j \geq 0$$

Then, in-queue, we have the average number:

$$E[N_j] = \frac{\rho_j}{1-\rho_j}$$

By Little's Law and the average response time is getting. For example, we have λ as external arrival rate, and we have the average response time formula:

$$E[R] = \frac{1}{\lambda}\sum_j E[N_j]$$

Poisson may not be the arrival process for a queue in feedback networks.

The following is just one simple example. Find a queue in which r constant μ is sent back to the same queue after a customer is served with a probability p very close to 1. If there is an absence, it is very likely that there will be an absence early and with a high chance the customer will be sent out again.

But when there is no customer in the network, since r is very small, it is highly unlikely that a n arrival will occur early. Apparently the cycle of arrival isn't memoryless. So the entire cycle of arrival may not be Poisson and the queue is not M / M/1.

Nonetheless, Jackson's Theorem also holds even though Poisson is not the complete cycle of arrival at each queue.Also, Jackson's Theorem can be extended to even more general scenarios, such as M / M / m queues. In order to allow the service rate at each queue to depend on the number of customers at that queue, we can generalize M / M / m or M / M / b. Suppose the service time at the jth queue is distributed exponentially at the rate $\mu_j(m)$, where m is the number in the queue just before the departure of the customer. We define the

$$\rho_j(m) = \frac{\lambda_j}{\mu_j(m)}, j =, \dots, K, m = 1,2, \dots$$

and

$$\hat{P}_j(N_j) = \begin{cases} 1, & N_j = 0 \\ \rho_j(1)\rho_j(2)\dots\rho_j(N_j) & N_j > 0 \end{cases}$$

We have:

Jackson's Theorem for State-Dependent Service Rates. We have $N_1, \dots, N_K \geq 0$ for all,

$$P(N_1, \dots,) = \frac{\hat{P}_1(N_1)\dots\hat{P}_k(N_k)}{G}$$

assuming $0 < G < \infty$,where G is the normalization factor:

 ISSN: 2278-4632
Vol-10 Issue-6 No. 13 June 2020

$$G = \sum_{N_1=0}^{\infty} \dots \dots \sum_{N_k}^{\infty} \hat{P}_1(N_1) \dots \hat{P}_k(N_k)$$

we need to change the device criteria for closed queueing networks by

$$\sum_{j=1}^{k} P_{ij} = 1, \quad i = 1, \dots, k \qquad (1)$$

And

$$\lambda_j = \sum_{i=1}^{k} \lambda_i P_{ij} \qquad (2)$$

We need to change the system criterion for closed queuing networks by remembering that there is no external entry into the system. Using the form

$$\lambda_j(M) = \alpha(M)\bar{\lambda}_j, \qquad j = 1, \dots, k \qquad (3)$$

Denote

$$\rho_j(m) = \frac{\bar{\lambda}_j}{\mu_j(m)} \qquad (4)$$

$$\hat{P}_j(N_j) = \begin{cases} 1, & N_j = 0 \\ \rho_j(1)\rho_j(2) \dots \dots \rho_j(N_j) & N_j > 0 \end{cases} \qquad (5)$$

And

$$G(M) = \sum_{\{(N_1,\dots,N_k)|N_1+\dots+N_k=M\}} \hat{P}_1(N_1) \dots \hat{P}_k(N_k) \qquad (6)$$

We have:

Jackson's Theorem for Closed Networks: can be solved under some conditions: we have for all $N_1, \dots,, N_k \geq 0$, and $N_1 + \dots + N_k = M$

$$P(N_1, \dots, N_k) = \frac{\hat{P}_1(N_1) \dots \hat{P}_k(N_k)}{G(M)} \qquad (7)$$

How many states has that system? Or how many non-negative integer solutions are possible for the $N_1 + \dots + N_k = M$? A small counting theory gives the result

$$\text{Number of system states} = \binom{M + K - 1}{M} \qquad (8)$$

This number increases exponentially with M and K, the calculation of G(M) with (6) is very difficult. Simple algorithms were designed to allow for this mission. If every device node is a single queue $\rho_j(m) = \rho_j$ for any m. (6) becomes

ISSN: 2278-4632
Vol-10 Issue-6 No. 13 June 2020

$$\mathrm{G(M)} = \sum_{\{(\mathrm{N_1},\ldots,\mathrm{N_k})|\mathrm{N_1}+\cdots+\mathrm{N_k}=\mathrm{M}\}} \rho_1^{\mathrm{N_1}} \ldots \rho_k^{\mathrm{N_k}} \qquad (9)$$

From (9), define a polynomial in z

$$\Gamma(z) = \prod_{i=1}^{k} \frac{1}{1-\rho_i z}$$

$$(1+\rho_1 z+{\rho_1}^2 z^2+\cdots)(1+\rho_2 z+{\rho_2}^2 z^2+\cdots)\ldots \qquad (10)$$

This is the generating function of G(1) , G(2) ,…

$$\Gamma(z) = \sum_{n=0}^{\infty} G(n) Z^n \qquad (11)$$

Where G (0) =1

Define

$$\Gamma_i(z) = \prod_{j=0}^{i} \frac{1}{1-\rho_j z}, \qquad j = 1, \ldots\ldots, K \qquad (12)$$

And

$$\Gamma_i(z) = \sum_{j=0}^{\infty} G_i(j) z^j, \qquad i = 1, \ldots, k \qquad (13)$$

Where $G_k(j) = G(j)$.

We will be able to get the recursive formula to compute $G_k(j)$:

$$G_i(j) = \; G_{i-1}(j) + \rho_i G_i, \begin{matrix} i = 2,3, \ldots . K \\ j = 1,2, \ldots . M \end{matrix} \qquad (14)$$

with the initial values $G_1(j) = {\rho_1}^j$, j =1, 2,...,M and $G_i(0) = 1$, i =1, 2,...,K .

This algorithm is efficient both in time and space.

Classification of queueing network’s based on the capacity of the components queues.

1) No blocking if all queues in the network are of infinite capacity.
2) Blocking May arises if one or more queues in the network are of finite capacity.

Some common Blocking models:

Consider a job which finishes service at Q_i and is then required to move Q_j, blocking may occur if Qj is a finite capacity queue and all its waiting positions are full, the blocking model then decides the action that will be taken to handle this as far as the blocked job is concerned.

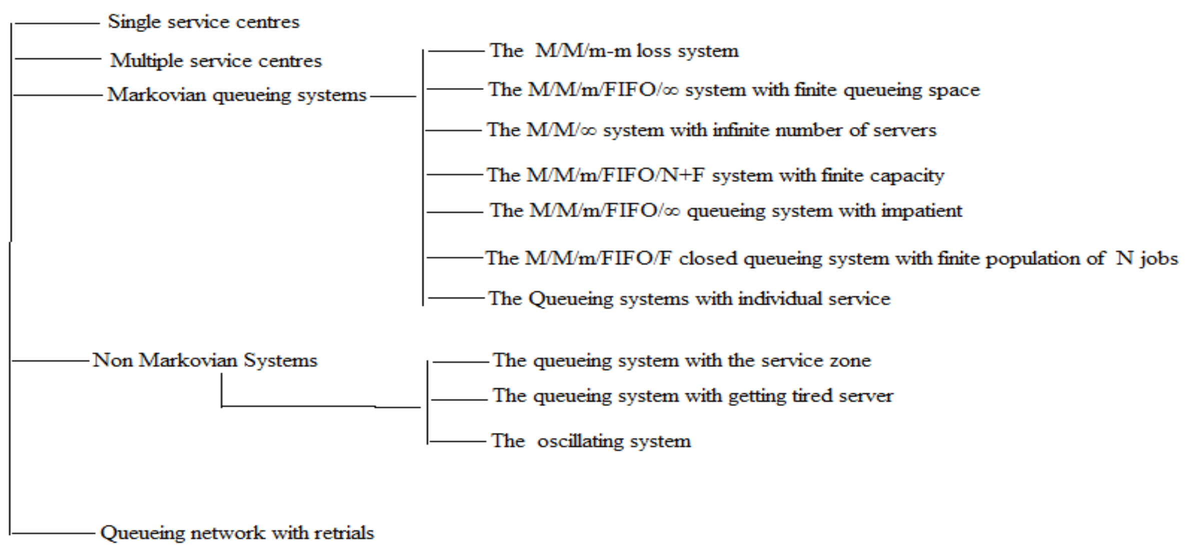
ISSN: 2278-4632
Vol-10 Issue-6 No. 13 June 2020

1) Rejection blocking – consider the situation where a job finishing service at Q_i wants to move to Q_j where Q_j is full, under rejection blocking, the blocked job is forced to leave the system.
2) Transfer blocking- As the situation where a job finishing service at Q_i wants to move to Q_j where Q_j is full Under Transfer Blocking, the blocked job waits at Q_i until Q_j is able to accept it.
3) Repetitive service Blocking – As the situation where a job finishing service at Q_i wants to move to Q_j where Q_j is full. Under repetitive service blocking, the blocked job goes for another service at Q_i and the process is repeated until the job can move out from Q_i.

For this case, two cases are usually called repetitive service with random destination, or repetitive service with set destination.

4) Blocking before service- before starts the service at Q_i, Q_j refers to destination queue in a job declares. Q_i gets blocked at the server, if Q_j is full at the entire instant, i.e. so it cannot be serve other jobs. Service to the job starts at Q_i only when the destination node Q_j gets unblocked and can accept the job after it finishes its service at the source queue Q_i.

Models and methodologies:



Find a network that consists of nodes and node-links.

Through applying an approximation of Kleinrock independence, each relation can be modelled as an M / M/1 queue. So we've got

The Average number of packets in queue or connection service (i, j) is

$$N_{ij} = \frac{\lambda_{ij}}{\mu_{ij} - \lambda_{ij}}$$

Sum of the packets average for all connections

$$N = \sum_{(i,j)} \frac{\lambda_{ij}}{\mu_{ij} - \lambda_{ij}}$$

When we know the total amount of traffic or demand entering in to the network, then by applying Little's Law, let's call it γ and disregard the delay in processing and propagation, the average delay per

$$T = \frac{1}{\gamma} \sum_{(i,j)} \frac{\lambda_{ij}}{\mu_{ij} - \lambda_{ij}}$$

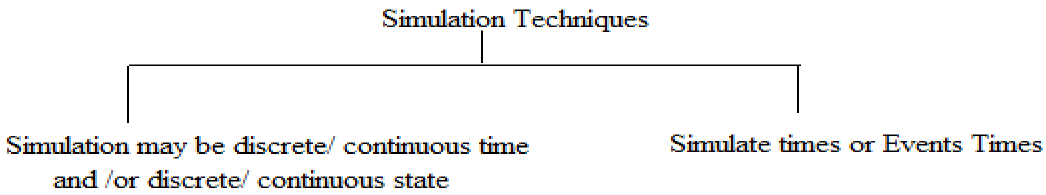
Where γ is the system's cumulative rate of arrival. If you cannot disregard the delay *ij d, you* can change the formula to

$$T = \frac{1}{\gamma} \sum_{(i,j)} \left(\frac{\lambda_{ij}}{\mu_{ij} - \lambda_{ij}} + \lambda_{ij} d_{ij}\right)$$

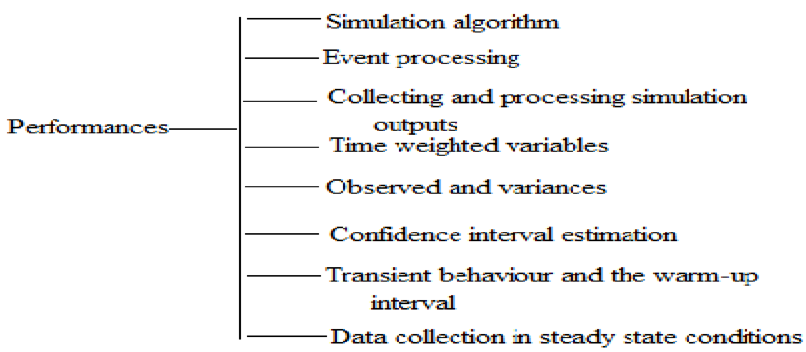
And the average delay per packet for a certain amount of traffic going through *p* is

$$T_p = \sum_{(i,j)\in p} \left(\frac{\lambda_{ij}}{\mu_{ij}(\mu_{ij} - \lambda_{ij})} + \frac{1}{\mu_{ij}} + d_{ij}\right)$$

Simulation Techniques for queueing networks:



Performance of Queueing Networks:



ISSN: 2278-4632
Vol-10 Issue-6 No. 13 June 2020

Performance measures: A cyclic queue is characterized by the following congestion problem:

M service centres are linked together in series such that the output of the last centre is the input to the first one; there are N items in this closed or cyclic system being served by each of the M centres in turn such a system may characterize the missing of ones, maintenance of busy period operations q_f transition system, etc.
one finds the following problems in applications of this theory:

1) Service rate is uncertain.
2) The configuration of the hypothesized does not fit in with actual models. Take an example of local train ticket counters here.

Mean value algorithm: The algorithm of great importance for networks of product type is the Mean Value Analysis (MVA)[18, 16]. This is an iterative algorithm developed to obtain the mean values of the output measurements without the constant of normalization being measured. The simple equation of MVA, if a network has K-1 jobs, relates the mean response time at system with K jobs and the mean number of workers at that system.
Unfortunately, the MVA calls for a lot of computational time. There are also approximation m ethods based on the MVA in the literature to quantify the output metrics, e.g. Bard-Schweitzer approximation (BS) used for single-server systems[18] or Self Correcting Approximation Technique (SCAT)[18, 19]. The results can be obtained using the following recursive process. Consider a closed network with m nodes containing w_{max} jobs. Each node has a single exponential server with mean service rate $1/\mu_i$, for i = 1, · · · ,m, and relative arrival rates given by the m-dimensioned vector **r** determined. The following algorithm can be used to obtain the mean waiting times for each node.

1. Set $W_k(1) = 1/\mu_k$ for k = 1, · · · ,m and set w = 2.
2. Determine $W_k(w)$ for k = 1, · · · ,m by

$$W_k(w) = \frac{1}{u_k}\left[1 + \frac{(w-1)r_k w_k(w-1)}{\sum_{j=1}^{m} r_j w_j(w-1)}\right]$$

3. If w = wmax, stop; otherwise, increment w by 1 and return to Step 2.

The results can be collected as the following recursive method Start of the recursion:

$\bar{N}_i[0] = 0$ In an empty network all mean queue lengths are zero.

Recursive steps:

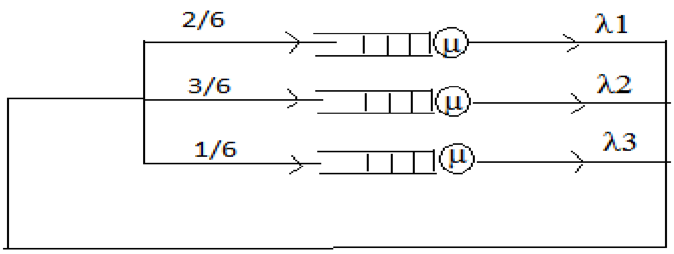
ISSN: 2278-4632
Vol-10 Issue-6 No. 13 June 2020

$$\overline{T}_i[k] = (1 + \overline{N}_i[k-1])\frac{1}{\mu_i}$$

$$\overline{N}_i[k] = k.\frac{\hat{\lambda}_i \overline{T}_i[k]}{\sum_{j=1}^{M} \hat{\lambda}_j \overline{T}_j[k]}$$

$$\lambda_i[k] = \frac{\overline{N}_i[k]}{\overline{T}_i[k]}$$

In the equation is the middle, the $\hat{\lambda}_i$ are any solution to the equations $\lambda_i = \sum_j \hat{\lambda}_j q_j, i$.



K= 6, A solution to the flow equations is $\hat{\lambda}_1 = 2, \hat{\lambda}_2 = 3, \hat{\lambda}_3 = 1$.

Starting from the initial values $\overline{N}_1[0] = \overline{N}_2[0] = \overline{N}_3[0] = 0$.One solves the mean values of progressively.

	λ_1			λ_2			λ_3		
K	T_1	N_1	λ_1	T_2	N_2	λ_2	T_3	N_3	λ_3
1	$\frac{1}{\mu}$	$\frac{1}{3}$	$\frac{\mu}{3}$	$\frac{1}{\mu}$	$\frac{1}{2}$	$\frac{\mu}{2}$	$\frac{1}{\mu}$	$\frac{1}{6}$	$\frac{\mu}{6}$
2	$\frac{4}{3\mu}$	$\frac{2}{3}$	$\frac{\mu}{2}$	$\frac{3}{2\mu}$	1	$\frac{2\mu}{3}$	$\frac{7}{6\mu}$	$\frac{1}{3}$	$\frac{2\mu}{7}$
3	$\frac{5}{3\mu}$	1	$\frac{3\mu}{5}$	$\frac{2}{\mu}$	$\frac{3}{2}$	$\frac{3\mu}{4}$	$\frac{4}{3\mu}$	$\frac{1}{2}$	$\frac{3\mu}{8}$
4	$\frac{2}{\mu}$	$\frac{4}{3}$	$\frac{2\mu}{3}$	$\frac{5}{2\mu}$	2	$\frac{4\mu}{5}$	$\frac{3}{2\mu}$	$\frac{2}{3}$	$\frac{4\mu}{9}$
5	$\frac{7}{3\mu}$	$\frac{5}{3}$	$\frac{5\mu}{7}$	$\frac{3}{\mu}$	$\frac{5}{2}$	$\frac{5\mu}{6}$	$\frac{5}{3\mu}$	$\frac{5}{6}$	$\frac{\mu}{2}$
6	$\frac{8}{3\mu}$	2	$\frac{3\mu}{4}$	$\frac{7}{2\mu}$	3	$\frac{6\mu}{7}$	$\frac{11}{6\mu}$	1	$\frac{6\mu}{11}$

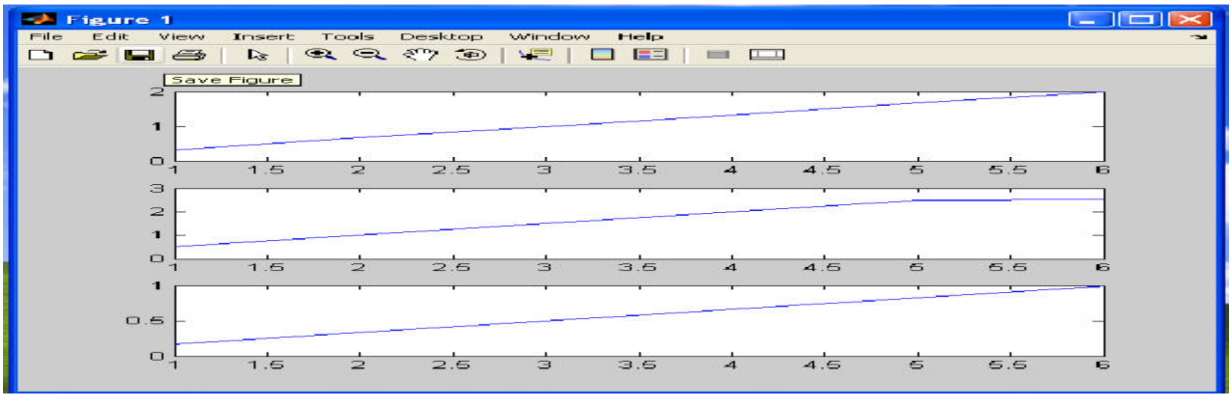


Figure 1: The mean number of customers

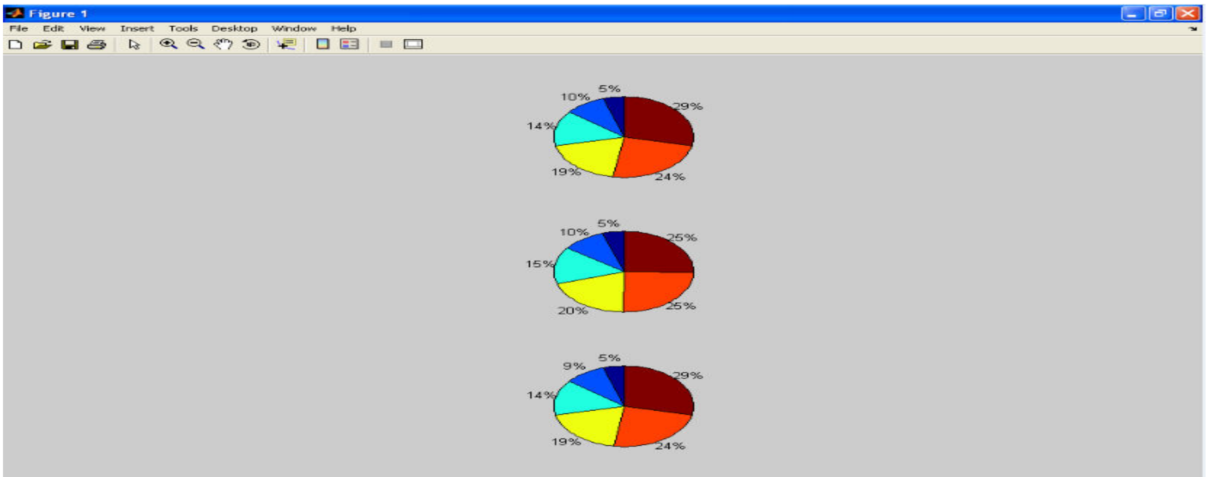


Figure 2: The mean number of customers by pie

Conclusion: In this paper, we have served the main theoretical results for queueing network. We also investigate the many applications and methodology of queueing networks etc. This queueing network literature is so rich which help to another basic researchers, we also refer to some earlier survey papers, books, and recent special issues for the survey on results of cyclic network queues. The recursive mean value analysis is providing variation of the arrivals rate when service rate is missing. The mean occupancy is increased when i is increased. We calculate the mean so journey time in queue; where the mean occupancy seen by a customer arriving at queue i, finally calculate the absolute values of customer streams through different queue. We see that the mean occupancy is increased when k is increased.

References:

[1]. Jerry Banks, John S. Carson II, Barry L Nelson, David M. Nicol. Discrete-Event System Simulation, Fourth Edition, Prentice Hall.

[2]. S. Stidham, Analysis, design and control of queueing systems, Operation Research 50(1), 197-216(2002).

[3]. G. Bolch, S. Greiner, H. De Meer, and K.S. Trivedi, Queueing Networks and Markov Chains. Modelling and Performance Evaluation with Computer Science Applications, John Wiley&Sons, Inc., Landon, 1998.

[4]. T. Czachorski, Queueing Models in Performance Evaluation of Computer Networks and Systems, Jacek Skalmierski's computer workshop, Gliwice, 1999.

[5]. B. Filipowicz, Stochastic Models in Operation Research: Analysis and Synthesis of the queueing systems and Networks, WNT, Warszawa, 1996.

[6]. B. Filipowich, Modelling and Optimization of queueing systems. Volume 1, Markovian systems, Krakow, 1999.

[7]. B. Filipowich, Modelling and Optimization of queueing systems. Volume 1, Markovian systems, Krakow, 1999.

[8]. L. Kleinrock, Queueing Systems, Volume 1, Theory, John Wiley & Sons, New York, 1975.

[9]. K. Idzikowska, Structural optimization of M/Mm/FIFO/m+N queueing system with individual service and flux of arrivals, ZN AGH Electrotechnics and Electronics 19 (1), 38-44(2000).

[10]. B. Filipowich, Modelling and Optimization of Queueing Systems. Volume 2, Non-Markovian systems, Krakow, 2000.

[11]. A. Rutkowska, The Models of Queueing Systems and Networks with Weibull Servers, Ph.D. Dissertation, AGH-UST, Krakow, 2001.

[12]. W. Weibull, A statistical distribution of wide applicability, J. Appl. Mech. 18, 293-297(1951).

[13]. H.C. Tijms, Stochastic models, An Algorithmic Approach, John Wiley & Sons, London, 1994.

[14]. A. Chydzinski, The M/G-G/1 oscillating queueing system, Queueing Systems 42 (3), 255-268(2002).

[15]. B. Filipowich and J. Kwiecien, Fork-Join Systems, ZN AGH Automatics 7 (3), 707-716(2003).

[16]. M. Reiser and S. Lavenberg, Mean value analysis of closed multichain queueing networks", J. ACM 27 (2), 313-322(1980).

[17]. F. Baskett, K. Chandy k, R. Muntz, and F. Palacios, Open, Closed and mixed networks of queues with different classes of customers", J. ACM 22(2), 248-260(1975).

[18]. G. Bolch, S. Greiner, H. de Meer, and K.S. Trivedi, Queueing Networks and Markov chains. Modelling and Performance Evaluation with Computer Science Applications, London, 1998.

[19] D. Neuse and K. Chandy, "SCAT: a heuristic algorithem for queueing network models of computing system", ACM sigmetrics Performance Evaluation Review 10(1), 59-79(1981).