On Nano Biclosure Space

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Abstract: Nano Biclosure space is an extension of Nano closure space. The purpose of this paper is to introduce and study**Nano Closure space and Nano Biclosure Space** and some of its properties.

Keywords: Nano closure space, Nano Biclosure space, Nano open sets, Nano closed sets, Nano Biclosure subspace.

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1. Introduction:

Mathematical relations are very useful in several fields such as, rough set theory, digital topology, biochemistry etc. Topology generated by relations is narrowing the gap between topologist and applications of topology.

M. Lellis Thivagar and Carmel Richard [5] introduced Nano topological space with respect to a subset X of a universe U which is defined in terms of lower and upper approximation of approximation space (U,R).Nasef A. A., Aggour A. I., Darwesh S. M. [3] also studied the Nano Topological space and explained some useful properties. Čech closure space was first introduced by E. Čech [1,2].Thivagar et.al [4] introduced a new topology called Čech Nano topology in terms of Čech rough closure operator.

In this paper we are introducing Nano closure space and Nano Biclosure space with examples and alsostudied some of its properties.

2. Preliminaries:

Definition 2.1: - [1]A function C: P(X) → P(X), where P(X) is a power set of a set X, is called a Čech closure operator for X provided the following conditions are satisfied:

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(1) $C(\emptyset) = \emptyset$, (2) $A \subset C(A)$ for each $A \subset X$, (3) $C(A \cup B) = C(A) \cup C(B)$ for each $A,B \subset X$.

Then the pair (X,C), where X is a non-empty set and C is a Čech closure operator for X, is called a Čech closure space. If (X, C) is a Čech closure space and $A \subset X$, then C(A) is called the closure of A in (X,C). The Čech closure space (X,C) is said to be Kuratowski(topological) space, if $C(C(A)) = C(A)$ for each $A \subset X$.

Definition 2.2[5]:- Let U be a non-empty set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements that belong to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lowerapproximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x \in U}\{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(2) The upper approximation of X with respects to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x \in U}\{R(x): R(x) \cap X \neq \emptyset\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and is denoted by $B_R(X)$. Thatis, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.3[4]:- Let P(X) be the power set of a rough set X in the approximation space (U,R). A function $C: P(X) \to P(X)$ is called a Čech rough closure (simply, rough closure) operator for X if it satisfy the following conditions:(1) $C(\emptyset) = \emptyset$, (2) $A \subset C(A)$ for each $A \subset X$.

(3) $C(A \cup B) = C(A) \cup C(B)$ for each $A,B \subset X$. Then the rough set X together with the Čech rough closure operator C is called a Čech rough closure space (simply, rough closure space) and it is denoted by (X,C).

3. Nano Closure space

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Definition 3.1:- Let U be a non-empty finite set of objects called the Universe and R be an equivalence relation on U and $X \subseteq U$ .Then Nanoclosure operator is a function $Ncl_R : P(X) \to P(X)$ such that for all

$$Ncl_R(A) = \begin{cases} \cup L_{iR}(X); if A \subseteq \cup L_{iR}(X) \\ \cup B_{iR}(X); if A \subseteq \cup B_{iR}(X) \\ X; otherwise \\ \phi; if A = \phi \end{cases} \quad \text{for all } A \subseteq X$$

Where L_i's are elements of $L_R(X)$ and B_i's are elements of $B_R(X)$.

Which satisfies three conditions:

1. $Ncl_R(\phi) = \phi$
2. $A \subseteq Ncl_R(A)$
3. $Ncl_R(A \cup B) = Ncl_R(A) \cup Ncl_R(B)$ Hence (X, Ncl_R) is called **Nano closure space**.

Example 3.2:-Consider the following table giving information about the germination of different kind of seeds banana (S_1), sugarcane (S_2), chilly(S_3), cotton(S_4),corn(S_5), groundnut (S_6), rose (S_7), paddy (S_8) depend on the necessary environmental factors say temperature, soil, water, sunlight.

	Heat	Type of Soil	Water	Sunlight	Germination
S_1	Moderate	Sand	Medium	Moderate	High
S_2	Moderate	Red	Medium	Low	Low
S_3	Moderate	Hard	Heavy	Low	High
S_4	High	Lose soil	Heavy	High	Low
S_5	High	Lose soil	Medium	High	Low
S_6	Moderate	Red	Medium	Moderate	High
S_7	High	Red	Medium	High	Low
S_8	High	Lose soil	Medium	High	High

Here universal set U= $\{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$ is a set of different seeds.

Here are two attributes- condition attributes and decision attributes (germination quality).

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C= {Heat, Type of soil, Water, Sunlight} is a set of condition attributes.

The entries in the table are the attribute values. Last column is decision attributes.

Consider the set X having high germination X= $\{S_1, S_3, S_6, S_8\}$.

Equivalent classes for set of condition attributes for **high germination**are

U/R=$\{\{S_1\}, \{S_2\}, \{S_3\}, \{S_4\}, \{S_5, S_8\}, \{S_6\}, \{S_7\}\}$

The upper and lower approximations of X with respect to Rare given by

$U_C(X) = \{\{S_1\}, \{S_3\}, \{S_5, S_8\}, \{S_6\}\}$, $L_C(X) =\{\{S_1\}, \{S_3\}, \{S_6\}\}$, $B_C(X) = \{\{S_5, S_8\}\}$

Consider Nano closure operator $Ncl_R : P(X) \to P(X)$ such that

$$N\tau_{cl_R}\{S_1\} = \{S_1\}, N\tau_{cl_R}\{S_3\} = \{S_3\}, N\tau_{cl_R}\{S_6\} = \{S_6\}, N\tau_{cl_R}\{S_8\} = \{S_5, S_8\},$$
$$N\tau_{cl_R}\{S_1, S_3\} = \{S_1, S_3\}, N\tau_{cl_R}\{S_1, S_6\} = \{S_1, S_6\},$$
$$N\tau_{cl_R}\{S_1, S_8\} = X = N\tau_{cl_R}\{S_3, S_8\} = N\tau_{cl_R}\{S_6, S_8\}$$
$$N\tau_{cl_R}\{S_1, S_3, S_6\} = \{S_1, S_3, S_6\},$$
$$N\tau_{cl_R}\{S_3, S_6, S_8\} = X = N\tau_{cl_R}\{S_1, S_3, S_8\} = N\tau_{cl_R}\{S_1, S_6, S_8\}$$
$$N\tau_{cl_R}\{\phi\} = \{\phi\}, N\tau_{cl_R}\{X\} = \{X\}.$$

Hence (X, Ncl_R) is a Nano closure space

Nano open sets of (X, Ncl_R)=$\{\{S_8\}, \{S_1, S_8\}, \{S_3, S_8\}, \{S_6, S_8\},\{S_3, S_6, S_8\}, \{S_1, S_3, S_8\}, \{S_1, S_6, S_8\}\}$.

Nano closed sets of (X, Ncl_R) = $\{\{S_1\}, \{S_3\}, \{S_6\},\{S_1, S_3\}, \{S_1, S_6\}, \{S_3, S_6\}, \{S_1, S_3, S_6\}\}$

Example 3.3: -Now consider a set X for lowGermination X= $\{S_2, S_4, S_5, S_7\}$.

Equivalent classes for set of condition attributes for low germination

U/R= $\{\{S_1\}, \{S_2, S_6\}, \{S_3\}, \{S_4, S_5\}, \{S_7\}, \{S_8\}\}$

The upper and lower approximations of X with respect to R are given by

$U_C(X) = \{\{S_2, S_6\}, \{S_4, S_5\}, \{S_7\}\}$, $L_C(X) = \{\{S_4, S_5\}, \{S_7\}\}$, $B_C(X) = \{\{S_2, S_6\}\}$

Consider Nano closure operator $Ncl_R : P(X) \to P(X)$ such that

$$
\begin{aligned}
&Ncl_R\{S_2\} = X, Ncl_R\{S_4\} = \{S_4, S_5\}, Ncl_R\{S_5\} = \{S_4, S_5\}, Ncl_R\{S_7\} = \{S_7\},\\
&Ncl_R\{S_2, S_4\} = X, Ncl_R\{S_2, S_5\} = X, Ncl_R\{S_4, S_5\} = \{S_4, S_5\}\\
&Ncl_R\{S_4, S_7\} = \{S_4, S_5, S_7\} = Ncl_R\{S_5, S_7\}, Ncl_R\{S_2, S_7\} = X\\
&Ncl_R\{S_2, S_4, S_5\} = X = Ncl_R\{S_2, S_5, S_7\},\\
&Ncl_R\{S_3, S_6, S_8\} = X = Ncl_R\{S_1, S_3, S_8\} = Ncl_R\{S_1, S_6, S_8\},\\
&Ncl_R\{\phi\} = \{\phi\}, Ncl_R\{X\} = \{X\}.
\end{aligned}
$$

Hence (X, Ncl_R) is Nano closure space.

Nano open sets of $(X, Ncl_R) = \{\{S_2\}, \{S_4\}, \{S_5\}, \{S_2, S_4\}, \{S_2, S_5\}, \{S_2, S_7\}, \{S_4, S_7\},$

$\{S_5, S_7\}, \{S_2, S_4, S_5\}, \{S_2, S_5, S_7\}, \{S_2, S_4, S_7\}\}$

Nano closed sets of (X, Ncl_R) = $\{\{S_7\}, \{S_4, S_5\}, \{S_4, S_5, S_7\}$

4. Nano Biclosure space

Definition 4.1:- Let U be a non-empty finite set of objects called the universe and R_1 and R_2 be two equivalence relations on U and X⊆U where i={1,2}.Then Nano closure operator Ncl_{R_i} is a function defined as $Ncl_{R_i} : P(X) \to P(X)$ where i={1,2} such that for A⊆X

$$
Ncl_{R_i}(A) = \begin{cases} \cup L_{iR}(X); if A \subseteq \cup L_{iR}(X) \\ \cup B_{iR}(X); if A \subseteq \cup B_{iR}(X) \\ X; otherwise \\ \phi; if A = \phi \end{cases}
$$

L_{iR}'s and B_{iR}'s are elements of $L_R(X)$, $B_R(X)$ respectively.

Which satisfies three conditions:

1.$Ncl_1(\phi) = \phi, Ncl_2(\phi) = \phi$

2.$A \subseteq Ncl_{R_1}(A), A \subseteq Ncl_{R_2}(A)$

3.$Ncl_{R_1}(A \cup B) = Ncl_{R_1}(A) \cup Ncl_{R_1}(B), Ncl_{R_2}(A \cup B) = Ncl_{R_2}(A) \cup Ncl_{R_2}(B)$

That is (X, Ncl_{R_1}) and (X, Ncl_{R_2}) are two Nano closure spaces.

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Hence $(X, Ncl_{R_1}, Ncl_{R_2})$ **is called Nano Biclosure space.**

Note 4.2: -

1. Nano open sets of Nano Biclosure space are open in both Nano closure spaces.
2. A subset A of a Nano Biclosure space $(X, Ncl_{R_1}, Ncl_{R_2})$ is called Nano closed if $Ncl_{R_1}(Ncl_{R_2} A) = A$.

   The complement of Nano closed set is called Nano open.
3. A is a Nano closed subset of Nano Biclosure space $(X, Ncl_{R_1}, Ncl_{R_2})$ if and only ifA is both Nano closed subset of (X, Ncl_{R_1}) and (X, Ncl_{R_2}).
4. Let A be a Nano closed subset of a Nano Biclosure space $(X, Ncl_{R_1}, Ncl_{R_2})$.

The following conditions are equivalent

1. $Ncl_{R_2}(Ncl_{R_1} A) = A$
2. $Ncl_{R_1} A = A$, $Ncl_{R_2} A = A$.

Remark 4.3. Let A be a subset of a Nano Biclosure space $(X, Ncl_{R_1}, Ncl_{R_2})$.

If A is a Nano open set in $(X, Ncl_{R_1}, Ncl_{R_2})$, then $Ncl_{R_1} Ncl_{R_2}(X - A) = Ncl_{R_2} Ncl_{R_1}(X - A)$.

The converse of the statement need not be true.

Example 4.4:- Consider example-1 as a Nano closure space (X, Ncl_{R_1}).

Now consider another equivalent class for set of condition attributes after removing one

Condition attribute “Type of soil” for high germination is

$U/R_2 = \{\{S_1, S_6\}, \{S_2\}, \{S_3\}, \{S_4\}, \{S_5, S_7, S_8\}\}$

Consider the set X having high germination X= $\{S_1, S_3, S_6, S_8\}$

The upper and lower approximations of X with respect to R_2 are given by

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$U_C(X) = \{\{S_1, S_6\}, \{S_3\}, \{S_5, S_7, S_8\}, L_C(X) = \{\{S_1, S_6\} \{S_3\}\}, B_C(X) = \{\{S_5, S_7, S_8\}\}$

Consider Nano closure operator $Ncl_{R_2} : P(X) \to P(X)$ such that

$$Ncl_{R_2}\{S_1\} = \{S_1, S_6\}, Ncl_{R_2}\{S_3\} = \{S_3\}, Ncl_{R_2}\{S_6\} = \{S_1, S_6\}, Ncl_{R_2}\{S_8\} = \{S_5, S_7, S_8\},$$
$$Ncl_{R_2}\{S_1, S_3\} = \{S_1, S_3, S_6\}\}, Ncl_{R_2}\{S_3, S_6\} = \{S_1, S_3, S_6\}, Ncl_{R_2}\{S_6, S_8\} = X = Ncl_{R_2}\{S_1, S_8\}$$
$$Ncl_{R_2}\{S_1, S_6\} = \{S_1, S_6\}, Ncl_{R_2}\{S_3, S_8\} = X,$$
$$Ncl_{R_2}\{S_1, S_3, S_6\} = \{S_1, S_3, S_6\}, Ncl_{R_2}\{S_3, S_6, S_8\} = X = Ncl_{R_2}\{S_1, S_3, S_8\} = Ncl_{R_2}\{S_1, S_6, S_8\}$$
$$Ncl_{R_2}\{\phi\} = \{\phi\}, Ncl_{R_2}\{X\} = \{X\}.$$

Nano open sets of $(X, Ncl_{R_2}) = \{\{S_1\}, \{S_3\}, \{S_6\}, \{S_8\}, \{S_1, S_3\}, \{S_3, S_6\}, \{S_1, S_8\}, \{S_3, S_8\},$

$\{S_3, S_6, S_8\}, \{S_1, S_3, S_8\}, \{S_1, S_6, S_8\}\}$.

Nano closed sets of (X, Ncl_{R_2}) = $\{\{S_1, S_6\}, \{S_1, S_3, S_6\}\}$

Here (X, Ncl_{R_2}) is a Nano closure space.

Hence $(X, Ncl_{R_1}, Ncl_{R_2})$ is Nano Biclosure space.

Nano open sets of $(X, Ncl_{R_1}, Ncl_{R_2}) = \{\{S_8\}, \{S_3, S_8\}, \{S_6, S_8\}, \{S_3, S_6, S_8\}, \{S_1, S_3, S_8\}, \{S_1, S_6, S_8\}\}$.

Nano closed sets of $(X, Ncl_{R_1}, Ncl_{R_2})$ = $\{\{S_1, S_6\}, \{S_1, S_3, S_6\}\}$

5. Properties of Nano Biclosure space

Proposition 5.1: - Let $(X, Ncl_{R_1}, Ncl_{R_2})$ be a Nano Biclosure space and let $A \subseteq X$ .Then

1. A is Nano open if and only if $A = X - Ncl_{R_1} Ncl_{R_2}(X - A)$.
2. If G is Nano open and $G \subseteq A$, then $G \subseteq X - Ncl_{R_1} Ncl_{R_2}(X - A)$.

Proof: -From above example,

1. Let A= $\{S_3, S_8\}$ is a Nano open set of $(X, Ncl_{R_1}, Ncl_{R_2})$.

X= $\{S_1, S_3, S_6, S_8\}$ X-A= $\{S_1, S_6\}$, $Ncl_{R_2}\{S_1, S_6\} = \{S_1, S_6\}$ and $Ncl_{R_1}\{S_1, S_6\} = \{S_1, S_6\}$

Now $X - \{S_1, S_6\} = \{S_3, S_8\}$ =A

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2. Let $G=\{S_8\}$ so $G\subseteq A=\{S_3, S_8\}$

From (1) part $X-Ncl_{R_1}Ncl_{R_2}(X-A)=A$ which shows that $G\subseteq X-Ncl_{R_1}Ncl_{R_2}(X-A)$.

Preposition 5.2:-Let (X,Ncl_{R_1},Ncl_{R_2}) is a Nano Biclosure space. If A and B are two Nanoclosed subsets of (X,Ncl_{R_1},Ncl_{R_2}).Then $A\cap B$ is also Nano closed.

Preposition 5.3:- Let (X,Ncl_{R_1},Ncl_{R_2}) is a Nano Biclosure space and Ncl_{R_1} and Ncl_{R_2} beadditive. If A and B are two Nano closed subsets of (X,Ncl_{R_1},Ncl_{R_2}).Then $A\cup B$ is alsoNano closed.

Proof: -Let A and B are two Nano closed subsets of (X,Ncl_{R_1},Ncl_{R_2}).Since Ncl_{R_1} and Ncl_{R_2} are said to satisfy additive property. Then $Ncl_{R_1}(Ncl_{R_2}(A))=A$ and $Ncl_{R_1}(Ncl_{R_2}(B))=B$.

Now consider, $$Ncl_{R_1}(Ncl_{R_2}(A\cup B))=Ncl_{R_1}(Ncl_{R_2}(A)\cup Ncl_{R_2}(B))=Ncl_{R_1}(Ncl_{R_2}(A))\cup Ncl_{R_1}(Ncl_{R_2}(B))$$
$$=A\cup B$$

Therefore $A\cup B$ is Nano closed if Ncl_{R_1} and Ncl_{R_2} are additive.

Lemma 5.4: -For any binary relations R_1 and $R_2\subseteq X\times X$ on X. (X,Ncl_{R_1},Ncl_{R_2}) is a Nano Biclosure space.

Proof:

1. $Ncl_{R_1}(\phi)=\phi and Ncl_{R_2}(\phi)=\phi$ i.e. $Ncl_{R_i}(\phi)=\phi$

2. For any subset A$\subseteq$X, A$\subseteq$Ncl$_R$ (A) by definition of Nano closure operator. i.e. $A\subseteq Ncl_{R_i}(A)$

3. To prove $Ncl_{R_i}(A\cup B)=Ncl_{R_i}(A)\cup Ncl_{R_i}(B)$

Since A and B are subsets of X then $A\cup B\subseteq X$,For every subset of X, by definition of Nano closure operator $Ncl_{R_1}(A\cup B)=Ncl_{R_1}(A)\cup Ncl_{R_1}(B)$, $Ncl_{R_2}(A\cup B)=Ncl_{R_2}(A)\cup Ncl_{R_2}(B)$

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By definition of Nano Biclosure space, above condition is also satisfied for $(X, Ncl_{R_1}, Ncl_{R_2})$ i.e.

$Ncl_{R_i}(A \cup B) = Ncl_{R_i}(A) \cup Ncl_{R_i}(B)$.

Proposition 5.5. If $(Y, Ncl_{R_3}, Ncl_{R_4})$ is a Nano Biclosure subspace of $(X, Ncl_{R_1}, Ncl_{R_2})$, then for every Nano open subset G of $(X, Ncl_{R_1}, Ncl_{R_2})$, G ∩ Y is an Nano open set in $(Y, Ncl_{R_3}, Ncl_{R_4})$.

Proof. Let G be aNano open set in $(X, Ncl_{R_1}, Ncl_{R_2})$. G is Nano open in both (X, Ncl_{R_1}) and (X, Ncl_{R_2}). Thus, $Ncl_{R_j}(Y-(G \cap Y)) = Ncl_{R_i}(Y-(G \cap Y)) \cap Y \subseteq Ncl_{R_i}(X-G) \cap Y = (X-G) \cap Y = Y-(G \cap Y)$ for each i ∈ {1,2}, j∈ {3,4}. Consequently, G ∩ Y is Nano open in both (Y, Ncl_{R_3}) and (Y, Ncl_{R_4}). Therefore, G ∩ Y is Nano open in $(Y, Ncl_{R_3}, Ncl_{R_4})$.

6. Nano Biclosure subspace

Definition 6.1:-Let $(X, Ncl_{R_1}, Ncl_{R_2})$ be a Nano Biclosure space. A Nano Biclosure space $(Y, Ncl_{R_3}, Ncl_{R_4})$ is called a Nano Biclosure subspace of $(X, Ncl_{R_1}, Ncl_{R_2})$ if $Y \subseteq X$ and $Ncl_j(A) = Ncl_i(A) \cap Y$ for each $i \in \{1,2\}$, $j = \{3,4\}$ and each subset $A \subseteq Y$.

Example 6.2:-Consider example-1 as a Nano closure space (X, Ncl_{R_1}).

Now consider another equivalent class for set of condition attributes after removing one Condition attribute "Type of soil" for high germination is

$U/R_2 = \{\{S_1, S_6\}, \{S_2\}, \{S_3\}, \{S_4\}, \{S_5, S_7, S_8\}\}$

Consider the set X having high germination $X = \{S_1, S_3, S_6, S_8\}$

The upper and lower approximations of X with respect to R_1 are given by

$U_C(X) = \{\{S_1, S_6\}, \{S_3\}, \{S_5, S_7, S_8\}\}$, $L_C(X) = \{\{S_1, S_6\} \{S_3\}\}$, $B_C(X) = \{\{S_5, S_7, S_8\}\}$

Consider Nano closure operator $Ncl_{R_2} : P(X) \to P(X)$ such that

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$Ncl_{R_2}\{S_1\}=\{S_1,S_6\}, Ncl_{R_2}\{S_3\}=\{S_3\}, Ncl_{R_2}\{S_6\}=\{S_1,S_6\}, Ncl_{R_2}\{S_8\}=\{S_5,S_7,S_8\},$
$Ncl_{R_2}\{S_1,S_3\}=\{S_1,S_3,S_6\}\}, Ncl_{R_2}\{S_3,S_6\}=\{S_1,S_3,S_6\}, Ncl_{R_2}\{S_6,S_8\}=X=Ncl_{R_2}\{S_1,S_8\}$
$Ncl_{R_2}\{S_1,S_6\}=\{S_1,S_6\}, Ncl_{R_2}\{S_3,S_8\}=X,$
$Ncl_{R_2}\{S_1,S_3,S_6\}=\{S_1,S_3,S_6\}, Ncl_{R_2}\{S_3,S_6,S_8\}=X=Ncl_{R_2}\{S_1,S_3,S_8\}=Ncl_{R_2}\{S_1,S_6,S_8\}$
$Ncl_{R_2}\{\phi\}=\{\phi\}, Ncl_{R_2}\{X\}=\{X\}.$

Nano open sets of (X, Ncl_{R_2}) = {{S_1}, {S_3}, {S_6}, {S_8}, {S_1, S_3}, {S_3, S_6}, {S_1, S_8}, {S_3, S_8},

{S_3, S_6, S_8}, {S_1, S_3, S_8}, {S_1, S_6, S_8}}

Nano closed sets of (X, Ncl_{R_2}) = {{S_1, S_6}, {S_1, S_3, S_6}}

Here (X, Ncl_{R_2}) is a Nano closure space.Hence $(X, Ncl_{R_1}, Ncl_{R_2})$ is a Nano Biclosure space.

Nano open sets of $(X, Ncl_{R_1}, Ncl_{R_2})$ are {{S_8}, {S_1, S_8}, {S_3, S_8}, {S_6, S_8}, {S_3, S_6, S_8},

{S_1, S_3, S_8}, {S_1, S_6, S_8}}

Nano closed sets of $(X, Ncl_{R_1}, Ncl_{R_2})$ are{{S_1, S_6}, {S_1, S_3, S_6}}

Consider another set Y having high germination with moderate temperature condition is

Y= {S_1, S_3, S_6}after removing “Type of soil” attribute.Here Y⊆X.

Consider equivalent classes U/R_3= {{S_1, S_6}, {S_2}, {S_3}, {S_4}, {S_5, S_7, S_8}}

Here lower and upper approximations are

U_C(Y) = {{S_1, S_6}, {S_3}}, L_C(Y) = {{S_1, S_6} {S_3}}, B_C(Y) = {∅}

Consider Nano closure operator $Ncl_{R_3}: P(Y) \to P(Y)$ such that

$Ncl_{R_3}\{S_1\}=\{S_1,S_6\}, Ncl_{R_3}\{S_3\}=\{S_3\}, Ncl_{R_3}\{S_6\}=\{S_1,S_6\},$
$Ncl_{R_3}\{S_1,S_3\}=\{S_1,S_3,S_6\}\}, Ncl_{R_3}\{S_3,S_6\}=\{S_1,S_3,S_6\},$
$Ncl_{R_3}\{S_1,S_6\}=\{S_1,S_6\},$
$Ncl_{R_3}\{\phi\}=\{\phi\}, Ncl_{R_3}\{Y\}=\{Y\}.$

Here (Y, Ncl_{R_3}) is a Nano closure space.

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Nano open sets of (Y, Ncl_{R_3}) are {{{S1}, {S6}, {S1, S3}, {S3, S6}}.

Nano closed sets of (Y, Ncl_{R_3}) are {{{S3}, {S1, S6}}.

Consider another class of relations after removing heat attribute for high germination for set

Y= {S1, S3, S6}, U/R4= {{S1}, {S2}, {S3}, {S4}, {S5, S8}, {S6}{S7}}

Here lower and upper approximations are

U_C(Y) = {{S1}, {S3}, {S6}}, L_C(Y) = {{S1}, {S3}, {S6}}, B_C(Y) = {Ø}

Consider Nano closure operator $Ncl_{R_4} : P(Y) \to P(Y)$ such that

$$Ncl_{R_4}\{S_1\} = \{S_1\}, Ncl_{R_4}\{S_3\} = \{S_3\}, Ncl_{R_4}\{S_6\} = \{S_6\},$$
$$Ncl_{R_4}\{S_1, S_3\} = \{S_1, S_3\}, Ncl_{R_4}\{S_3, S_6\} = \{S_3, S_6\},$$
$$Ncl_{R_4}\{S_1, S_6\} = \{S_1, S_6\},$$
$$Ncl_{R_4}\{\phi\} = \{\phi\}, Ncl_{R_4}\{Y\} = \{Y\}.$$

Here (Y, Ncl_{R_4}) is a Nano closure space.Nano open sets of (Y, Ncl_{R_4}) are {Y, Ø}

Nano closed sets of (Y, Ncl_{R_4}) are $\{\{S_1\},\{S_3\},\{S_6\},\{S_1,S_3\},\{S_3,S_6\},\{S_1,S_6\}\}$

Hence $(Y, Ncl_{R_3}, Ncl_{R_4})$ is a Nano Biclosure space.

Nano open sets of $(Y, Ncl_{R_3}, Ncl_{R_4})$ are {Y, Ø}

Nano closed sets of $(Y, Ncl_{R_3}, Ncl_{R_4})$ are $\{\{S_3\},\{S_1,S_6\}\}$

Here $(Y, Ncl_{R_3}, Ncl_{R_4})$ is a Nano Biclosure subspace of $(X, Ncl_{R_1}, Ncl_{R_2})$ whereY⊆X.

Proposition 6.3:- Let $(X, Ncl_{R_1}, Ncl_{R_2})$ be a Nano Biclosure space and let $(Y, Ncl_{R_3}, Ncl_{R_4})$ be a Nano Biclosure subspace of $(X, Ncl_{R_1}, Ncl_{R_2})$. If F is a Nano closed subset of $(Y, Ncl_{R_3}, Ncl_{R_4})$, then F is also a Nano closed subset of $(X, Ncl_{R_1}, Ncl_{R_2})$.

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Proof: - Let F be a Nano closed subset of $(Y, Ncl_{R_3}, Ncl_{R_4})$. Then $Ncl_{R_3}(F) = F$ and $Ncl_{R_4}(F) = F$. Since Y is Nano closed subset of both (X, Ncl_{R_1}) and (X, Ncl_{R_2}), $Ncl_{R_1}(F) = F$ and $Ncl_{R_2}(F) = F$ Consequently, F is both a Nano closed subset of and (X, Ncl_{R_2}). Therefore, F is a fuzzy closed subset of $(X, Ncl_{R_1}, Ncl_{R_2})$.

7. **Conclusion: -**In this paper the concept of Nano closure space and Nano Biclosure space has introduced. Some of its properties have been proved.

8. **References: -**

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