# "PROBABILISTIC S-METRIC SPACE AND PROPOSED SOME FIXED POINT RESULTS"

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**Abstract**
In this paper, we introduce new notions of generalized Probabilistic S-metric space. After elementary observations we will discuss to investigate and prove basic properties of fixed point theorems in new notions of generalized space which extend the previously generalized many results of great mathematicians. In fact, a probabilistic metric space is a generalization of metric space defined as where the distance has no longer values in non-negative real numbers, but in distribution functions. A probability distribution function is some function that may be used to define a particular probability distribution function**.**

**Keyword:** Probabilistic S-Metric Space.

## 1. Introduction and preliminaries

The fixed point theory in metric spaces has been attracted and subject of study over six decades due to numerous applications in areas such as variational and linear inequalities, optimization, and approximation theory. In 1942, Menger [1] introduced the notion of probabilistic metric space (briefly PM-space) as a generalization of metric space. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physical quantities and physiological thresholds. The development of fixed point theory in PM-spaces was studied by Schweizer and Sklar [2, 3]. Fixed point theory has been always an interesting area of research since 1922 with the Banach contraction fixed point theorem. Different type of generalizations of metric space have been investigated by Gähler [4, 5] and Dhage also [6, 7] also Ha et al [8].Concept of generalized metric space was introduced [9],[10],[11].New authors have studied of Dhages notion of D metric space[12].Some fixed point results investigated in probabilistic metric space [13][14][15][16][17][18]. Probabilistic G-metric space introduced by M. Janfada, A. R. Janfada and Z. Mollaee [21]

**Definition 1.1** Let $f: X \to X$ be a map. Then an element $x \in X$ is said to be **fixed point** of $f$ if $f(\mathrm{x}) = x$.

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**Definition 1.2** [22] Distribution function is a function $F_{p,q}$ is associated with every pair of points p and q non empty set S, rather than negative integer. distribution

Function $F$ maps extended reals $\bar{\mathrm{R}} := \mathrm{R} \cup \{-\infty, +\infty\}$ into the closed interval [0, 1], is left-continuous at every real point, non-decreasing and satisfies $F(-\infty) = 0$ and $F(+\infty) = 1$. The set of all distribution functions is denoted by $\Delta$ .the value of $F_{p,q}(x)$ for $x \geq 0$ may then be considered as probability that distance between p and q is (strictly) less than x.

**Distance distribution functions 1.3** [22] they form the subset $\Delta_+ \subset \Delta$ of those distribution functions that satisfy
$F(0)=0$, $\Delta_+ := \{ F \in \Delta : F(0) = 0\}$
The set of all the distribution functions is denoted by $\Delta$ and the set of those distribution functions such that $F(0) = 0$ is denoted by $\Delta^+$.
A natural ordering in $\Delta$ is defined by $F \leq G$ whenever $F(x) \leq G(x)$, for every $x \in \mathrm{R}$.
The maximal element in this order for $\Delta^+$ is $\varepsilon_0$, where for $-\infty \leq a \leq \infty$ the distribution function $\varepsilon_{a(x)}$ is defined as 0 if $-\infty \leq x \leq a$ and 1 if $a < x \leq \infty$.

**Definition 1.4** [22][23] A **triangular norm** $\tau$ is a binary operation on $\Delta_+$ that is commutative, associative, non decreasing in each of its variables and has $\varepsilon_0$ as the identity. A binary operation on $\Delta$ that is associative, commutative, non decreasing and whose restriction to $\Delta_+$ is a triangle function is called a multiplication.

**Definition 1.5** [22][23] A **triangular norm** (briefly a *t* **-norm**) is a binary operation $T$ on the unit interval (0, 1) that is asso- ciative, commutative, non-decreasing in each of its variables and such that $T(x, 1) = x$ for every $x \in [0, 1]$.

Examples of $t$-norms have all a probabilistic meaning, $W(x, y) := \max\{0, x + y - 1\}$, $\Pi(x, y) := x\,y$ *and* $M(x, y) := \min\{x,y\}$.

**Definition 1.6** [1] A probabilistic metric space (PM space) is an ordered triple $(S,F,\tau)$ where S is a non empty set, $\tau$ is a triangle function and $F$ is a map (the **probabilistic distance**) from $S \times S$ into $\Delta+$ , and the following holds,

(a) $F(p, q) = \varepsilon_0$ if and only if, $p = q$;
(b) If $p \neq q$ , then $F(p, q) \neq \varepsilon_0$
(c) for all $p, q \in S$, $F(p, q) = F(q, p)$;
(d) for all $p, q, r \in S$, $F(p, r) \geq \tau(F(p, q), F(q, r))$.

Mustafa and Sims [19] introduced the notion of a G-metric space. Recently in 2012 Sedghi et al. [20] introduced a new generalized metric space called an S-metric space.

 

**Definition 1.7** [20] **S-metric space:** Let $X$ *be* a nonempty set, An $S$*-metric on* $X$ *is* a function $S : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,

(1) $S(x, y, z) \geq 0$,

(2) $S(x, y, z) = 0$ if and only if $x = y = z$,

(3) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$ The pair $(X, S)$ is called an *S-metric space.*

A sequence $\{x_n\}$ in $X$ converges to $x$ if and only if $S(x_n, x_n, x) \to 0$ as $n \to \infty$. That is for each $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $S(x_n, x_n, x) < \varepsilon$

and we denote this by $\lim_{n\to\infty} x_n = x$. A sequence $\{x_n\}$ in $X$ is called a *Cauchy sequence* if for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $S(x_n, x_n, x_m) < \varepsilon$ for each $n, m \geq n_0$.
The $S$-metric space $(X, S)$ is said to be *complete* if every Cauchy sequence is convergent,

## 2. Main Result

We extend now the new generalization of S-Metric space with the probabilistic approach. We introduce the Probabilistic S-metric space defined as;
**Probabilistic S-Metric Space** Let X is a nonempty set, $\tau$ is a triangle function and $S: X\times X\times X \to \Delta^+$, is a mapping satisfies the following,

1. $S(p,p,p) = \varepsilon_0$
2. If $p \neq q$ then $S(p,p,q) \neq \varepsilon_0$
3. if $S(p,q,r) \geq \tau((S(p,p,s), S(q,q,s), S(r,r,s))$

   for all $p,q,r,s \in X$. Then $(X,S,\tau)$ is called a generalized probabilistic S-metric space.

The Probabilistic S-metric space is called proper if $\tau(\varepsilon_a, \varepsilon_b) \geq \epsilon_{a+b}$ for all $a,b \in [0, \infty)$

## 3. Research Methodology

The idea behind the investigation of this new generalized Probabilistic S-metric space is probabilistic approach and the treatment of probability distribution function. A probabilistic metric space is a generalization of metric space defined as where the distance has no longer values in non-negative real numbers, but in distribution functions. A probability distribution function is some function that may be used to define a particular probability distribution function.

Motivated by the generalization of this space we conclude that analytical research approach to be adopted for further investigations of new mappings of fixed point contraction. Ls mapping, Cs mapping, Ciric's fixed point theorems have been studied by Nihal Yilmaz Ozgur and Nihal Tag[24] . Several problems in pure and applied mathematics have as their solutions the fixed point of some mappings and number of procedures in numerical analysis. Our object in this paper to discuss about new generalized Probabilistic S-metric space and then investigate the results of fixed point applications in this metric space, also we establish some fixed point theorems in the Probabilistic S-metric space, which were given by great mathematicians. The Banach Contraction Principle is one of the cornerstones in the development of Nonlinear Analysis, in general, and metric fixed point theory, in particular. This principle was extended and improved in many directions and various fixed point theorems were established. Fixed point theorem helps to obtain the result of an error estimate for iterative scheme which is used to approximate the fixed point.

**4. Acknowledgement**

Obviously, We get inspired by new generalized Probabilistic S-metric space. Now I can extend the further investigation of contraction mappings in fixed point theory using the generalized notions of Probabilistic S-metric space in my PhD thesis and obtain new outcomes of fixed point results.

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