Tri-b connectedness in tritopological space

Ranu Sharma [1,a] ***and*** **Sachin Sharma**[2,b]

[1,a]Department of Applied Mathematics and Computational Science, SGSITS, Indore(M.P.), India,

[1,b]Department of Mathematics, SJHSG Innovative College, Indore(M.P.), India,

Email: ranusamarth@gmail.com, sachin.150880@gmail.com

Abstract.

The aim of this paper is to introduce new type of tri-b connectedness in tri topological spaces and also defined tri-b separation properties in tri topological spaces.

Keywords: tri-b connectedness, tri-b disconnectedness and tri-b separation.

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1. Introduction

The idea of b-open sets in a topological space was given by D. Andrijvic [4] in 1996. Al-Hawary & A.Al-omari [3] defined the notion of b-open set & b-continuity in bitopological space and established several fundamental properties. Abo Khadra and Nasef [2] discussed b-open set in bitopological spaces. Pervin W.J.[10] , I.L. Reilly [12] , J. Swart [14] , B. Dvalishi [6] and T. Birsan [5] studied connectedness in bitopological spaces . A. Kandil and others [1] studied connectedness in bitopological ordered spaces and in ideal bitopological spaces. Tri topological space is a generalization of bitopological space. The study of tri-topological space was first initiated by Martin M. Kovar[8]. S. Palanimmal [9] investigated tri toplogical spaces in 2011..N.F. Hameed & Mohammed Yahya Abid [7] studied separation axioms in tri-topological spaces and gives the definition of 123 open set in tri topological spaces. P. Priyadharsini and A. Parvathi [11] introduced tri-b open sets in tri topological spaces. R. Sharma B.A Deole and S. Verma[13] studied some properties of tri-b open sets.

In this paper, we introduce tri-b connectedness and tri-b separated sets in tri topological space. Here we are using tri-b open set in place of 123 open set.

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2. Preliminaries

Definition 2.1[9]: Let X be a nonempty set and T_1, T_2 and T_3 are three topologies on X. The set X together with three topologies is called a tri topological space and is denoted by (X, T_1, T_2, T_3).

Definition 2.3[11]: Let (X, T_1, T_2, T_3) be a tri topological space, a subset A of a space X is said to be tri-b open set if $S \subset tri-cl(tri-\text{int}\, S) \cup tri-\text{int}(tri-clS)$.

Definition 2.4[11]: Let (X, T_1, T_2, T_3) be a tri topological space and let $A \subset X$. The intersection of all tri-b closed sets containing A is called the tri-b closure of A & denoted by $tri-b-clA$. Tri-b-intA is the union of all tri-b open sets contained in A.

Definition 2.4[10]: A bitopological space is (X, T_1, T_2) said to be connected if and only if X cannot be expressed as the union of two non empty disjoint sets A and B such that A is T_1 open and B is T_2 open. When X can be so expressed, we write $X = A/B$ and call this a separation of X.

3. tri-b separated sets in tritopological space

Definition 3.1: Let (X, T_1, T_2, T_3) be a tri topological space, two non empty subsets A and B of X are said to be tri-b separated if and only if $A \cap tri-b\,cl(B) = \phi$ and $tri-b\,cl(A) \cap B = \phi$. These two conditions are equivalent to the single condition $[A \cap tri-b\; cl(B)] \cup [tri-b\; cl(A) \cap B] = \phi$

Example 3.2 : Let $X = \{a, b, c\}$, $T_1 = \{X, \phi\}$, $T_2 = \{X, \phi, \{a\}\}$, $T_3 = \{X, \phi, \{b, c\}\}$

Tri open sets in tri topological spaces are union of all tri topologies.

Then tri open sets of $X = \{X, \phi, \{a\}, \{b, c\}\}$

Tri-b open set of X is denoted by $tri-BO(X) = \{X, \phi, \{a, b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

If we take $A = \{a, b\}$ and $B = \{c\}$ Then

$[A \cap tri\,cl(B)] \cup [tri\,cl(A) \cap B] = \{\{b\} \cap \{d\}\} \cup \{\{b\} \cap \{d\}\} = \phi$.

Then $\{a,b\}$ and $\{c\}$ are tri-b separated sets.

Theorem 3.3 : If A and B are tri-b separated subsets of a tri topological space (X,T_1,T_2,T_3) and $C\subset A$ and $D\subset B$, then C and D are also tri-b separated .

Proof: Since A and B are tri-b separated sets then $A\cap tri-b\ cl(B)=\phi$ and

$tri-b\,cl(A)\cap B=\phi$ ……………..(1)

Also $C\subset A\Rightarrow tri-b\,cl(C)\subset tri-b\ \ cl(A)$ and

$D\subset B\Rightarrow tri-b\ cl(D)\subset tri-b\ cl(B)$ …………………(2)

By (1) and (2) that $C\cap tri-b\ cl(D)=\phi$ and $tri-b\ cl(C)\cap D=\phi$

Hence C and D are tri-b separated.

Theorem 3.4 : Two tri-b closed (tri-b open) subsets A , B of a tri topological space (X,T_1,T_2,T_3) are tri-b separated if and only if they are disjoint .

Proof : Since any two tri-b separated sets are disjoint, we have to prove that two disjoint tri-b closed (tri-b open) sets are tri-b separated .

If A and B are both disjoint tri-b closed, then :

$A\cap B=C,\ tri-b\ cl(A)=A$ and $tri-b\,cl(B)=B$ …..[1]

So that $tri-b\,cl(A)\cap B=\phi$ and $A\cap tri-b\ cl(B)=\phi$

Therefore A and B are tri-b separated.

If A and B are both disjoint and tri-b open , then A^c and B^c are both tri-b closed so that :

$tri-b\ cl(A^c)=A^c$ and $tri-b\ cl(B^c)=B^c$

Also $A\cap B=\phi\Rightarrow A\subset B^c$ and $B\subset A^c$

$\Rightarrow tri-b\ cl(A)\subset tri-b\ cl(B^c)=B^c$ and $tri-b\,cl(B)\subset tri-b\ cl(A^c)=A^c$

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$\Rightarrow tri-b\,cl(A)\cap B=\phi$ and $A\cap tri-b\,cl(B)=\phi$

Hence $\Rightarrow A$ and B are tri-b separated.

4. tri-b connected and tri-b disconnected sets in tri topological space

Definition 4.1 : Let (X,T_1,T_2,T_3) be a tri topological space , a subset A of X is said to be tri-b disconnected if and only if it is the union of two non empty tri-b separated sets . That is, if and only if there exist two non empty tri-b separated sets C and D such that $C\cap tri-b\,cl(D)=\phi$, $tri-b\ cl(C)\cap D=\phi$ and $A=C\cup D$, A is said to be tri-b connected if and only if it is not tri-b disconnected.

Example 4.2: Let $X=\{a,b,c\}$ **,** $T_1=\{X,\phi\}$ **,** $T_2=\{X,\phi,\{a\}\}$**,** $T_3=\{X,\phi,\{b,c\}\}$

Tri open sets in tri topological spaces are union of all tri topologies.

Then tri open sets of $X=\{X,\phi,\{a\},\{b,c\}\}$

Tri-b open set of X is denoted by $tri-BO(X)=\{X,\phi,\{a,b\},\{c\},\{a,c\},\{b,c\}\}$**.**

If we take $A=\{a,b,c\}$, $C=\{a,b\}$ and $D=\{c\}$ Then

$A=C\cup D$ and $\{a,b\}\cap\{c\}=\phi$ and

$C\cap tri-b\,cl(D)=\phi$, $tri\,cl(C)\cap D=\phi$.

Then $\{a,b\}$ and $\{c\}$ are tri-b separated sets.

Hence the set $\{a,b,c\}$ is tri-b disconnected.

Remarks 4.3 :

(i)The empty set in $tri-b\ O(X)$ is trivially tri-b connected.

(ii) Every singleton set in $tri-b\,O(X)$ is tri-b connected since it cannot be expressed as a union of two non empty tri-b separated sets.

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Theorem 4.4 : A tri topological space (X, T_1, T_2, T_3) is tri-b disconnected if and only if there exists a non empty proper subset of X which is both tri-b open and tri-b closed in X.

Proof : Let A be a non empty proper subset of X which is both tri-b open and tri-b closed in X . We have to show that X is tri-b disconnected :

Let $B = A^c$. Then B is non empty since A is a proper subset of X .Moreover , $A \cup B = X$ and $A \cap B = \phi$, since A is both tri-b open and tri-b closed , B is also both tri-b open and tri-b closed .

Hence $tri-b\ cl(A) = A$ and $tri-b\ cl(B) = B$ it follows that $tri-b\, cl(A) \cap B = \phi$ and $A \cap tri-b\ \ cl(B) = \phi$. Thus X has been expressed as a union of two tri-b separated sets and so X is tri-b disconnected.

Conversely: Let X is tri-b disconnected. Then there exist non empty subsets A and B of X such that $tri-b\ cl(A) \cap B = \phi$ and $A \cap tri-b\ cl(B) = \phi$ and $A \cup B = X$. Since $tri-b\ cl(A) = A$ and $tri-b\, cl(B) = B \Rightarrow A \cap B = \phi.$

Hence $A = B^C$ and B is non empty, A is a proper subset of X . Now $A \cup tri-b\, cl(B) = X$.

[since $A \cup B = X$ and $B \subset tri-b\, cl(B) \Rightarrow X \subset A \cup tri-b\, cl(B)$ but $A \cup tri\, cl(B) \subset X$ always]

Also $A \cap tri-b\ cl(B) = \phi \Rightarrow A = (tri-b\, cl(B))^C$ and similarly $B = (tri-b\, cl(A))^C$.

Since $tri-b\, cl(A)$ and $tri-b\, cl(B)$ are tri-b closed sets, it follows that A and B are tri-b open sets , and since $A = B^C$, A is also tri-b closed . Thus A is non empty proper subset of X which is both tri-b open and tri-b closed.

In the same way we can show that B is also non empty proper subset of X which is both tri-b open and tri-b closed.

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Theorem 4.5: Let (X, T_1, T_2, T_3) be a tri topological space and A and B be non empty sets in space X.

(i) If A and B are tri-b separated and $A_1 \subset A$ and $B_1 \subset B$, then A_1 and B_1 are so.

(ii) If $A \cap B = \phi$ such that each of A and B are both tri-b open (tri-b closed) ,then A and B are tri-b separated.

(iii) If each of A and B both tri-b open (tri-b closed) and if $H = A \cap (X - B)$ and $G = B \cap (X - A)$, then H and G.

Proof : (i) Since $A_1 \subset A$ then $tri-b clA_1 \subset tri-b\ clA$ .Then $B \cap tri-b cl(A) = \phi$ implies $B_1 \cap tri-b cl(A) = \phi$ and $B_1 \cap tri-b\ cl(A_1) = \phi$ .Similarly $A_1 \cap tri\, cl(B_1) = \phi$ .Hence A_1 and B_1 are tri-b separated.

(ii) Since $A = tri-b\ cl(A)$ and $B = tri-b cl(B)$ and $A \cap B = \phi$,then $tri-b\ cl(A) \cap B = \phi$ and $tri-b\ cl(B) \cap A = \phi$ .Hence A and B are separated. If A and B are tri-b open , then their complements are tri-b closed.

(iii) If A and B are tri-b open , then $X - A$ and $X - B$ are tri-b closed .Since $H \subset X - B$, $tri-b\ cl(H) \subset tri-b\ cl(X - B) = X - B$ and so $tri-b\ cl(H) \cap B = \phi$ .Thus $G \cap tri-b cl(H) = \phi$ .Similarly , $H \cap tri-b cl(B) = \phi$ .Hence H and G are tri-b separated .

Theorem 4.6: Let (X, T_1, T_2, T_3) be a tri topological space and $E \subset X$ .If E is tri-b connected, then so is $tri-b cl(E)$.

Proof : Let E be the tri-b connected subset of a tri topological space (X, T_1, T_2, T_3). To prove that $tri-b cl(E)$ is connected .Suppose contrary, Then $tri-b\ cl(E)$ is disconnected .Then their

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exist non empty sets $A, B \subset X$ such that $tri-b\,cl A \cap B = \phi,\ A \cap tri-b\,cl B = \phi$.

$tri-b\ cl\ E = A \cup B$

$A \cup B = tri-b\ cl\ E \supset E,$

$\Rightarrow E \subset A \cup B,\ E$ is connected .

$\Rightarrow E \subset A$ or $E \subset B$ {by theorem 4.5}

$E \subset A \Rightarrow tri-b\ cl\ E \subset tri-b\ cl\ A$

$\Rightarrow E \subset A \cup B,$
$\Rightarrow tri-b\ cl\ E \cap B \subset tri-b\,cl\ A \cap B = \phi$ .....(i)

$tri-b\,cl\ E = A \cup B$
$\Rightarrow B \subset tri-b\,cl\ E$
$\Rightarrow tri-b\,cl\ E \cap B = B$
$\Rightarrow B = \phi$

For $tri-b\,cl\ E \cap B = \phi$ (from (i))

H in X such that $tri-b\,cl\,(E) = G \cup H$ .Since $E = (G \cap E) \cup (H \cap E)$ and $tri-b\ cl(G \cap E) \subset tri-b\ cl(G)$ and $tri-b\,cl(H \cap E) \subset tri-b\,cl(H)$ and $G \cap H = \phi$ then $(tri-b\ cl(G \cap E)) \cap H = \phi$. Hence $(tri-b\ cl(G \cap E)) \cap (H \cap E) = \phi$ .Similarly $(cl(H \cap E)) \cap (G \cap E) = \phi$ .Therefore E is connected a contradiction for $B \neq \phi$ similarly $E \subset B \Rightarrow A = \phi$ .Again we get a contradiction .Hence , If E is tri-b connected, then so is $tri-b\ cl\,(E)$.

Corollary 4.7: A tri topological space (X, T_1, T_2, T_3) is tri-b connected if and only if the only non empty subset of X which is both tri-b open and tri-b closed in X is X itself .

Theorem 4.8: A tri topological space (X, T_1, T_2, T_3) is tri-b disconnected if and only if any one of the following statements holds :

(i) X is the union of two non empty disjoint tri-b open sets .

(ii) X is the union of two non empty disjoint tri-b closed sets .

Proof: Let X be a tri-b disconnected . Then there exists a nonempty proper subset A of X which is both tri-b open and tri-b closed . Then A^C is also both tri-b open and tri-b closed also $A \cup A^C = X$. Hence the sets A and A^C satisfy the requirements of (i) and (ii).

Conversely ; let $X = A \cup B$ and $A \cap B = \phi$, where A, B are non empty tri-b open sets . It follows that $A = B^C$ so that A is tri-b closed.

Since B is non empty, A is a proper subset of X . Thus A is a non empty proper subset of X which is both tri-b open and tri-b closed. Hence by the theorem(4.4) , X is tri-b disconnected .

Again , let $X = C \cup D$ and $C \cap D = \phi$, where C , D are non empty tri-b closed sets . then $C = D^C$ so that C is tri-b open . Since D is non empty , C is a proper subset of X which is both tri-b open and tri-b closed . Hence X is tri-b disconnected by the theorem. Thus it is shown that if any one of the conditions (i) and (ii) holds , then X is tri-b disconnected .

Corollary 4.9: A subset Y of a tri topological space X is tri-b disconnected if and only if Y is the union of two non empty disjoint sets both tri-b open (tri-b closed) in Y .

Theorem 4.10: Let (X, T_1, T_2, T_3) be a tri topological space .If A is a tri-b connected set of X and H, G are tri-b separated sets of X with $A \subset H \cup G$, then either $A \subset H$ or $A \subset G$.

Proof : Let $A \subset H \cup G$ .Since $A = (A \cap H) \cup (A \cap G)$, then $(A \cap G) \cap tri-b\ cl(A \cap H) \subset G \cap cl(H) = \phi$. By similar reasoning .we have $(A \cap H) \cap tri-b\ cl(A \cap G) \subset H \cap cl(G) = \phi$ .Suppose that $A \cap H$ and $A \cap G$ are nonempty. Then A is not tri-b connected. This is a contradiction. Thus either $A \cap H = \phi$ or $A \cap G = \phi$ .This implies that $A \subset H$ or $A \subset G$.

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Theorem 4.11: Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') are two tri topological space .Let $f: X \to Y$ be a continuous function. if M is tri-b connected in X ,then $f(M)$ is tri-b connected in Y .

Proof : Suppose that $f(M)$ is tri-b disconnected in Y .There exist two tri-b separated sets P and Q of Y such that $f(M) = P \cup Q$. Set $A = M \cap f^{-1}(P)$ and $B = M \cap f^{-1}(Q)$. Since $f(M) \cap P \neq \phi$ then $M \cap f^{-1}(P) \neq \phi$ and so $A \neq \phi$.Similarly $B \neq \phi$. Since $P \cap Q = \phi$, $A \cap B = M \cap f^{-1}(P \cap Q) = \phi$ and so $A \cap B = \phi$ .Since f is continuous then by Lemma **4.5** , $tri-b\ cl(f^{-1}(Q)) \subset f^{-1}(tri-b\ cl(Q))$ and $B \subset f^{-1}(Q)$ then $tri-b\,cl(B) \subset f^{-1}(tri-b\ cl(Q))$ .Since $P \cap tri-b\ cl(Q) = \phi$, then $A \cap f^{-1}(tri-b\,cl(Q)) \subset f^{-1}(P) \cap f^{-1}(tri-b\,cl(Q)) = \phi$ and then $A \cap tri-b\ cl(B) = \phi$. Thus A and B are tri-b separated.

Theorem 4.12: If A is a tri-b connected set of an tri topological space (X,T_1,T_2,T_3) and $A \subset B \subset tri-b\ \ cl(A)$ then B tri-b connected.

Proof: Suppose B is not tri-b connected. There exist tri-b separated sets U and V of X such that $B = U \cup V$ .This implies that U and V are nonempty and $tri-b\,cl(U) \cap V = U \cap tri-b\ cl(V) = \phi$. By Theorem 4.7, we have either $A \subset U$ or $A \subset V$. Suppose that $A \subset U$. Then $tri-b\,cl(A) \subset tri-b\,cl(U)$ and $V \cap tri-b\,cl(A) = \phi$ .This implies that $V \subset B \subset tri-b\ cl(A)$ and $V = tri-b\,cl(A) \cap V = \phi$. Thus, V is an empty set for if V is nonempty, this is a contradiction. Suppose that $A \subset V$. By similar way, it follows that U is empty. This is a contradiction. Hence, B is tri-b connected.

Theorem 4.13: If $\{M_i : i \in N\}$ is a nonempty family of tri-b connected sets of an tri topological space (X,T_1,T_2,T_3) with $\bigcap_{i \in I} Mi \neq \phi$ Then $\bigcup_{i \in I} Mi$ is tri-b connected.

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Proof: Suppose that $\bigcup_{i\in I} Mi$ is not tri-b connected. Then we have $\bigcup_{i\in I} Mi = H \cup G$, where H and G are tri-b separated sets in X. Since $\bigcap_{i\in I} Mi \neq \phi$ we have a point x in $\bigcap_{i\in I} Mi$ Since $x \in \bigcup_{i\in I} Mi$, either $x \in H$ or $x \in G$. Suppose that $x \in H$. Since $x \in Mi$ for each $i \in N$, then Mi and H intersect for each $i \in N$. By theorem 4.12; $Mi \subset H$ or $Mi \subset G$.Since H and G are disjoint, $Mi \subset H$ for all $i \in Z$ and hence $\bigcup_{i\in I} Mi \subset H$. This implies that G is empty. This is a contradiction. Suppose that $x \in G$. By similar way, we have that H is empty. This is a contradiction. Thus, $\bigcup_{i\in I} Mi$ is tri-b connected.

Theorem 4.14: If A and B are tri-b connected sets which are tri-b separated ,then $A \cup B$ is tri-b connected.

Proof: Suppose $A \cup B$ is tri-b disconnected and suppose $G \cup H$ is a disconnection of $A \cup B$ .Since A is a tri-b connected subsets of $A \cup B$, either $A \subset G$ or $A \subset H$ .Similarly either $B \subset G$ or $B \subset H$.

If $A \subset G$ and $B \subset H$ then $(A \cup B) \cap G = G$ and $(A \cup B) \cap H = H$ are tri-b separated but this contradicts the hypothesis. Hence either $A \cup B \subset G$ or $A \cup B \subset B$ and so $G \cup H$ is not a disconnection of $A \cup B$ .In other words $A \cup B$ is tri-b connected.

CONCLUSION:

In this paper the idea of tri-b connectedness and tri-b separation were introduced and studied in tri topological spaces.

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