A Result on Coupled Fixed Point Theorem in Complex Valued Partial b-Metric Space Using Contractive Conditions

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Abstract

The aim of this paper is to introduce the notion of a coupled fixed point result on complex partial b-metric space under contractive conditions. We furnish an example to validate the main theorem.

Keywords: common fixed point theorem, coupled fixed point, complex partial b-metric space, contractive condition.

1 Introduction

Fixed point theory is a useful in many branches of research in Mathematical science. Many branches of Pure and applied Mathematics the Banach contraction principle plays a major role is solving problems in this area. Azam et al.[5] introduced the new concept of complex valued metric space and he established many fixed point results. P.Dhivya and M.Marudai [6] obtained some new notion of complex partial metric and established the existence of common fixed point theorems satisfying a contractive condition of rational expression on a ordered complex partial metric space. Bhaskar, Lakshmikantham [10] and Ciri, Lakshmikantham [11] studied the existence and uniqueness of a coupled fixed point theorems on partially ordered metric space. Hassen Aydi [1] introduced coupled fixed point theorems and proved some coupled fixed point results for mapping satisfying different contraction condition on complete partial metric space. Recently, M. Gunaseelan [12] established some fixed point theorem by generating the contractive conditions in the context of complex partial b-metric space. The aim of this manuscript is to study the existence and uniqueness of coupled fixed point theorem on complex b-metric space under contractive condition.

2 Preliminaries

Let $\mathbb{C}$ be the set of complex numbers and $\lambda_1, \lambda_2 \in \mathbb{C}$. Define a partial order $\preceq$ on $\mathbb{C}$ as follows:

$$\lambda_1 \preceq \lambda_2 \text{ if and only if } \mathrm{Re}(\lambda_1) \le \mathrm{Re}(\lambda_2) \text{ } and \text{ } \mathrm{Im}(\lambda_1) \le \mathrm{Im}(\lambda_2) .$$

Consequently, one can infer that $\lambda_1 \preceq \lambda_2$ if one of the following conditions is satisfied:

i. $\mathrm{Re}(\lambda_1) = \mathrm{Re}(\lambda_2)$ *and* $\mathrm{Im}(\lambda_1) < \mathrm{Im}(\lambda_2)$,
ii. $\mathrm{Re}(\lambda_1) < \mathrm{Re}(\lambda_2)$ *and* $\mathrm{Im}(\lambda_1) = \mathrm{Im}(\lambda_2)$,
iii. $\mathrm{Re}(\lambda_1) < \mathrm{Re}(\lambda_2)$ *and* $\mathrm{Im}(\lambda_1) < \mathrm{Im}(\lambda_2)$,
iv. $\mathrm{Re}(\lambda_1) = \mathrm{Re}(\lambda_2)$ *and* $\mathrm{Im}(\lambda_1) = \mathrm{Im}(\lambda_2)$,

In particular, we will write $\lambda_1 \precnsim \lambda_2$ if $\lambda_1 \neq \lambda_2$ and one of *(i)*, *(ii)* and *(iii)* is satisfied and we will write $\lambda_1 \prec \lambda_2$ if only *(iii)* is satisfied. Notice that

a) If $0 \preccurlyeq \lambda_1 \precnsim \lambda_2$, then $|\lambda_1| < |\lambda_2|$,
b) If $\lambda_1 \preceq \lambda_2$ and $\lambda_2 \preceq \lambda_3$ then $\lambda_1 \preceq \lambda_3$.
c) If $a, b \in R$ and $a \le b$ then $a\lambda_1 \preccurlyeq b\lambda_1$ for all $\lambda \in \mathbb{C}$.

Definition 2.1. A complex partial b-metric on a non-void set X is a function $\varsigma_{cb} : X \times X \to \mathbb{C}^{+}$ such that for all $\lambda, \mu, \kappa, \nu \in X$.

i. $0 \preccurlyeq \varsigma_{cb}(\lambda, \lambda) \preccurlyeq \varsigma_{cb}(\mu, \lambda)$ (smallself-distance)
ii. $\varsigma_{cb}(\lambda, \mu) = \varsigma_{cb}(\mu, \lambda)$ (symmetry)
iii. $\varsigma_{cb}(\lambda, \lambda) = \varsigma_{cb}(\lambda, \mu) = \varsigma_{cb}(\mu, \mu) \Leftrightarrow \lambda = \mu$ (equality)
iv. $\exists$ a real number $s \ge 1$ such that $\varsigma_{cb}(\lambda, \mu) \preccurlyeq s[\varsigma_{cb}(\lambda, \kappa) + \varsigma_{cb}(\kappa, \mu)] - \varsigma_{cb}(\kappa, \kappa)$ (triangularity).

A complex partial b-metric space is a pair (X, ς_{cb}) such that X is a non-void set and ς_{cb} is complex partial b-metric on X. The number s is called coefficient of (X, ς_{cb}).

Remark 2.1. In a complex partial b-metric space (X, ς_{cb}) is $\lambda, \mu \in X$ and $\varsigma_{cb}(\lambda, \mu) = 0$, then $\lambda = \mu$, but the converse may not be true.

Remark 2.2. It is clear that every complex partial metric space is a complex partial b-metric space with coefficient $s = 1$ and every complex valued b-metric is a complex partial b-metric space with the same coefficient and zero self-distance. However, the converse of this fact need not hold.

Now, we define Cauchy sequence and convergent sequence in complex partial b-metric spaces.

Definition 2.2. [10] Let (X, ς_{cb}) be a complex partial b-metric space with coefficient s. Let $\{\lambda_n\}$ be any sequence in X and $\lambda \in X$. Then

i. The sequence $\{\lambda_n\}$ is said to be convergent with respect to τ_{cb} and converges to λ, if $\lim_{n\to\infty} \varsigma_{cb}(\lambda_n, \lambda) = \varsigma_{cb}(\lambda, \lambda)$.
ii. The sequence $\{\lambda_n\}$ is said to be Cauchy sequence in (X, ς_{cb}) if $\lim_{n,m\to\infty} \varsigma_{cb}(\lambda_n, \lambda_m)$ exists and is finite.
iii. (X, ς_{cb}) is said to be a complete complex partial b-metric space if for every Cauchy sequence $\{\lambda_n\}$ in X there exist $\lambda \in X$ such that $\lim_{n,m\to\infty} \varsigma_{cb}(\lambda_n, \lambda_m) = \lim_{n\to\infty} \varsigma_{cb}(\lambda_n, \lambda) = \varsigma_{cb}(\lambda, \lambda)$.
iv. A mapping $\xi : X \times X \to X$ is said to be continuous at $\lambda_0 \in X$ if for every $\varepsilon > 0$, there exists $\omega > 0$ such that $\xi(B_{\varsigma_{cb}}(\lambda_0, \omega)) \subset B_{\varsigma_{cb}}(\xi(\lambda_0, \varepsilon))$.

Let X be a complex partial b-metric space and $B \subseteq X$. A point $\lambda \in X$ is called an interior of set B, if there exists $0 < r \in \mathbb{C}$ such that $B_{\varsigma_{cb}}(\lambda, r) = \{\mu \in X : \varsigma_{cb}(\lambda, \mu) < \varsigma_{cb}(\lambda, \lambda) + r\} \subseteq B$. A subset B is called open, if each point of B is an interior point of B. A point $\lambda \in X$ is said to be a limit point of B, for every $0 < r \in \mathbb{C}$, $B_{\varsigma_{cb}}(\lambda, r) \cap (B - \{\lambda\}) \neq \phi$. A subset $B \subseteq X$ is called closed, B, for contains all its limit points.

Lemma 2.3. **[5]** Let (X, ς_{cb}) be complex partial b-metric space. A sequence $\{\lambda_n\}$ is Cauchy sequence in the CPBMS (X, ς_{cb}) then $\{\lambda_n\}$ is Cauchy in a metric space (X, ς_{cb}^t).

Definition 2.3. Let (X, ς_{cb}) be a complex partial b-metric space (CPBMS). Then an element $(\lambda, \kappa) \in X \times X$ is said to be a coupled fixed point of the mapping $\xi : X \times X \to X$ if $\xi(\lambda, \kappa) = \lambda$ and $\xi(\kappa, \lambda) = \kappa$.

3 Main Results

Theorem 3.1. Let (X, ς_{cb}) be complex partial b-metric space. Suppose that the mapping $\xi : X \times X \to X$ satisfies the following contractive condition for all $\lambda, \mu, \kappa, \nu \in X$

$$\varsigma_{cb}(\xi(\lambda, \mu), \xi(\kappa, \nu)) \leq \alpha \varsigma_{cb}(\lambda, \kappa) + \beta \varsigma_{cb}(\mu, \nu),$$

where α, β are nonnegative constant with $\alpha + \beta < 1$. Then, ξ has a unique coupled fixed point.

Proof. Choose $\lambda_0, \mu_0 \in X$ and set $\lambda_1 = \xi(\lambda_0, \mu_0)$ and $\mu_1 = \xi(\mu_0, \lambda_0)$.

Continuing this process, set $\lambda_{n+1} = \xi(\lambda_n, \mu_n)$ and $\mu_{n+1} = \xi(\mu_n, \lambda_n)$.

Then,

$$\begin{aligned}\varsigma_{cb}(\lambda_n, \lambda_{n+1}) &= \varsigma_{cb}(\ \xi(\lambda_{n-1}, \mu_{n-1})\ , \xi(\lambda_n, \mu_n)\ ) \\ &\leq \alpha\ \varsigma_{cb}(\lambda_{n-1}, \lambda_n) + \beta \varsigma_{cb}(\mu_{n-1}, \mu_n),\end{aligned}$$

which implies that

$$\left| \ \varsigma_{cb}(\lambda_n, \lambda_{n+1}) \right| \leq \alpha \left| \ \varsigma_{cb}(\lambda_{n-1}, \lambda_n) \right| + \beta \left| \ \varsigma_{cb}(\mu_{n-1}, \mu_n) \right| , \tag{1}$$

Similarly, one can prove that

$$\left| \ \varsigma_{cb}(\mu_n, \mu_{n+1}) \right| \leq \alpha \left| \ \varsigma_{cb}(\mu_{n-1}, \mu_n) \right| + \beta \left| \ \varsigma_{cb}(\lambda_{n-1}, \lambda_n) \right| , \tag{2}$$

From (1) and (2), we get

$$\left|\ \varsigma_{cb}(\lambda_n,\mu_{n+1})\ \right|+\left|\ \varsigma_{cb}(\mu_{bn},\mu_{n+1})\ \right|\le(\alpha+\beta)\left(\ \left|\ \varsigma_{cb}(\mu_{n-1},\mu_n)\ \right|+\left|\ \varsigma_{cb}(\lambda_{n-1},\lambda n)\ \right|\ \right)$$

$$=\rho\left(\ \left|\ \varsigma_{cb}(\mu_{n-1},\mu_n)\ \right|+\left|\ \varsigma_{cb}(\lambda_{n-1},\lambda_n)\ \right|\ \right)$$

where $\rho=\alpha+\beta<1$

Also,

$$\left|\ \varsigma_{cb}(\lambda_{n+1},\lambda_{n+2})\ \right|\le\alpha\left|\ \varsigma_{cb}(\lambda_n,\lambda_{n+1})\ \right|+\beta\left|\ \varsigma_{cb}(\mu_n,\mu_{n+1})\ \right|, \tag{3}$$

$$\left|\ \varsigma_{cb}(\mu_{n+1},\mu_{n+2})\ \right|\le\alpha\left|\ \varsigma_{cb}(\lambda_n,\mu_{n+1})\ \right|+\beta\left|\ \varsigma_{cb}(\mu_n,\lambda_{n+1})\ \right|, \tag{4}$$

From (3) and (4), we get

$$\left|\ \varsigma_{cb}(\lambda_{n+1},\lambda_{n+2})\ \right|+\left|\ \varsigma_{cb}(\lambda_{n+1},\mu_{n+2})\ \right|\le(\alpha+\beta)\left(\ \left|\ \varsigma_{cb}(\mu_n,\mu_{n+1})\ \right|+\left|\ \varsigma_{cb}(\lambda_n,\lambda_{n+1})\ \right|\ \right)$$

$$=\rho\left(\ \left|\ \varsigma_{cb}(\mu_n,\mu_{n+1})\ \right|+\left|\ \varsigma_{cb}(\lambda_n,\lambda_{n+1})\ \right|\ \right)$$

Repeating this way, we get

$$\left|\ \varsigma_{cb}(\lambda_n,\lambda_{n+1})\ \right|+\left|\ \varsigma_{cb}(\mu_n,\mu_{n+1})\ \right|\le\rho\left(\ \left|\ \varsigma_{cb}(\mu_{n-1},\mu_n)\ \right|+\left|\ \varsigma_{cb}(\lambda_{n-1},\lambda_n)\ \right|\ \right)$$

$$\le\rho^2\left(\ \left|\ \varsigma_{cb}(\mu_{n-2},\mu_{n-1})\ \right|+\left|\ \varsigma_{cb}(\lambda_{n-2},\lambda_{n-1})\ \right|\ \right)$$

$$\le\cdots\cdots\le\rho^n\left(\ \left|\ \varsigma_{cb}(\mu_0,\mu_1)\ \right|+\left|\ \varsigma_{cb}(\lambda_0,\lambda_1)\ \right|\ \right).$$

Now, if $\left|\ \varsigma_{cb}(\lambda_n,\lambda_{n+1})\ \right|+\left|\ \varsigma_{cb}(\mu_n,\mu_{n+1})\ \right|=\delta_n$, then

$$\delta_n\le\rho\delta_{n-1}\le\rho^2\delta_{n-2}\le\cdots\cdots\le\rho^n\delta_0. \tag{5}$$

If $\delta_0=0$ then $\left|\ \varsigma_{cb}(\lambda_0,\lambda_1)\ \right|+\left|\ \varsigma_{cb}(\mu_0,\mu_1)\ \right|=0$. Hence $\lambda_0=\lambda_1=\xi(\lambda_0,\mu_0)$ and $\mu_0=\mu_1$ $=\xi(\mu_0,\lambda_0)$, which implies that (λ_0,μ_0) is a coupled fixed point of ξ.

Let $\delta_0>0$. For each $n\ge m$, we have

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$$\varsigma_{cb}(\lambda_n,\lambda_m)\le s\ \ [\varsigma_{cb}(\lambda_n,\lambda_{n-1})+\varsigma_{cb}(\lambda_{n-1},\lambda_{n-2})]-\varsigma_{cb}(\lambda_{n-1},\lambda_{n-1})$$

$$\le s^2\ \ [\varsigma_{cb}(\lambda_{n-2},\lambda_{n-3})+\varsigma_{cb}(\lambda_{n-3},\lambda_{n-4})]-\varsigma_{cb}(\lambda_{n-3},\lambda_{n-3})$$

$$\le\cdots\cdots\le s^m\ \ [\varsigma_{cb}(\lambda_{n-2},\lambda_{n-3})+\varsigma_{cb}(\lambda_{n-3},\lambda_{n-4})]-\varsigma_{cb}(\lambda_{m+1},\lambda_{m+1})$$

$$\le\ s\ \ \varsigma_{cb}(\lambda_n,\lambda_{n-1})+s^2\ \varsigma_{cb}(\lambda_{n-1},\lambda_{n-2})+\cdots\cdots+s^m\ \varsigma_{cb}(\lambda_{m+1},\lambda_m).$$

which implies that

$$\left|\ \varsigma_{cb}(\lambda_n,\lambda_m)\ \right|\le s\ \left|\ \varsigma_{cb}(\lambda_n,\lambda_{n-1})\ \right|+s^2\left|\ \varsigma_{cb}(\lambda_{n-1},\lambda_{n-2})\ \right|+\cdots\cdots+s^m\ \left|\ \varsigma_{cb}(\lambda_{m+1},\lambda_m)\ \right|,$$

Similarly, one can prove that

$$\left|\ \varsigma_{cb}(\mu_n,\mu_m)\ \right|\le s\ \left|\ \varsigma_{cb}(\mu_n,\mu_{n-1})\ \right|+s^2\left|\ \varsigma_{cb}(\mu_{n-1},\mu_{n-2})\ \right|+\cdots\cdots+s^m\ \left|\ \varsigma_{cb}(\mu_{m+1},\mu_m)\ \right|,$$

Thus,

$$\left|\ \varsigma_{cb}(\lambda_n,\lambda_m)\ \right|+\left|\ \varsigma_{cb}(\mu_n,\mu_m)\ \right|\le s\delta_{n-1}+s^2\delta_{n-2}+\cdots\cdots+s^m\delta_m$$

$$\le\ \ [s\rho^{n-1}+s^2\rho^{n-2}+\cdots\cdots+s^m\rho^m]\delta_0$$

$$\le\frac{s\rho^m}{1-s\rho}\delta_0\to 0\qquad n\to\infty\,.$$

which implies that $\{\lambda_n\}$ and $\{\mu_n\}$ are Cauchy sequence in (X,ς_{cb}). Since the partial b-metric space (X,ς_{cb}) is complete, there exists $\lambda,\mu\in X$ such that $\{\lambda_n\}\to\lambda$ and $\{\mu_n\}\to\mu$ as $n\to\infty$ and $\varsigma_{cb}(\lambda,\lambda)=\lim\limits_{n\to\infty}\varsigma_{cb}(\lambda,\lambda_n)=\lim\limits_{n,m\to\infty}\varsigma_{cb}(\lambda_n,\lambda_m)=0$, $\varsigma_{cb}(\mu,\mu)=\lim\limits_{n\to\infty}\varsigma_{cb}(\mu,\mu_n)=\lim\limits_{n,m\to\infty}\varsigma_{cb}(\mu_n,\mu_m)=0$.

Now we have to prove that $\lambda=\xi(\lambda,\mu)$. We suppose on the contrary that $\lambda\ne\xi(\lambda,\mu)$ and $\mu\ne\xi(\mu,\lambda)$ so that $0<\varsigma_{cb}(\lambda,\xi(\lambda,\mu))=\alpha_1$ and $0<\varsigma_{cb}(\mu,\xi(\mu,\lambda))=\alpha_2$ then

$$\alpha_1=\varsigma_{cb}(\lambda,\xi(\lambda,\mu))\le\varsigma_{cb}(\lambda,\lambda_{n+1})+\varsigma_{cb}(\lambda_{n+1},\xi(\lambda,\mu))$$

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$$= \varsigma_{cb}(\lambda, \lambda_{n+1}) + \varsigma_{cb}(\xi(\lambda_n, \mu_n), \xi(\lambda, \mu))$$

$$\leq \varsigma_{cb}(\lambda, \lambda_{n+1}) + \alpha\ \varsigma_{cb}(\lambda_n, \lambda) + \beta\ \varsigma_{cb}(\mu_n, \mu)$$

which implies that

$$|\alpha_1| \leq \left|\ \varsigma_{cb}(\lambda, \lambda_{n+1})\ \right| + \alpha \left|\ \varsigma_{cb}(\lambda_n, \lambda)\ \right| + \beta \left|\ \varsigma_{cb}(\mu_n, \mu)\ \right|$$

As $n \to \infty$, $|\alpha_1| \leq 0$. Which is a contradiction, therefore $\left|\ \varsigma_{cb}(\lambda, \xi(\lambda, \mu))\ \right| = 0$

$\Rightarrow \quad \lambda = \xi(\lambda, \mu)$. Similarly we can prove that $\mu = \xi(\mu, \lambda)$. Thus (λ, μ) is a coupled fixed point of ξ. Now, if (u, v) is another coupled fixed point of ξ, then

$$\varsigma_{cb}(\lambda, u) = \varsigma_{cb}(\xi(\lambda, \mu), \xi(u, v)) \leq \alpha\ \varsigma_{cb}(\lambda, u) + \beta\ \varsigma_{cb}(\mu, v),$$

Thus,

$$\varsigma_{cb}(\lambda, u) \leq \frac{\beta}{1-\alpha} \varsigma_{cb}(\mu, v) \tag{6}$$

which implies that

$$\left|\ \varsigma_{cb}(\lambda, u)\ \right| \leq \frac{\beta}{1-\alpha} \left|\ \varsigma_{cb}(\mu, v)\ \right| \tag{7}$$

Similarly,

$$\left|\ \varsigma_{cb}(\mu, v)\ \right| \leq \frac{\beta}{1-\alpha} \left|\ \varsigma_{cb}(\lambda, u)\ \right| \tag{8}$$

From (9) and (8), we get

$$\left|\ \varsigma_{cb}(\lambda, u)\ \right| + \left|\ \varsigma_{cb}(\mu, v)\ \right| \leq \frac{\beta}{1-\alpha}\left(\ \left|\ \varsigma_{cb}(\lambda, u)\ \right| + \left|\ \varsigma_{cb}(\mu, v)\ \right|\ \right)$$

$$\left(1 - \frac{\beta}{1-\alpha}\right)\left(\ \left|\ \varsigma_{cb}(\lambda, u)\right| + \left|\ \varsigma_{cb}(\mu, v)\ \right|\ \right) \leq 0$$

Since $\alpha+\beta<1$, this implies that $\left|\ \varsigma_{cb}(\lambda,u)\right|+\left|\ \varsigma_{cb}(\mu,v)\right|\le 0$. Therefore $\lambda=u$ and

$\mu=v \quad \Rightarrow \quad (\lambda,\mu)=(u,v)$.

Thus, ξ has a unique coupled fixed point.

Corollary 3.2. Let (X,ς_{cb}) be a complete complex partial b-metric space. Suppose that the mapping $\xi: X\times X\to X$ satisfies the following contractive condition for all $\lambda,\mu,\kappa,\nu\in X$

$$\varsigma_{cb}(\xi(\lambda,\mu),\xi(\kappa,\nu))\le\frac{\alpha}{4}(\varsigma_{cb}(\lambda,\kappa)+\varsigma_{cb}(\mu,\nu)), \tag{9}$$

where $0\le\alpha<1$. Then, ξ has a unique coupled fixed point.

Example 3.3. Let $X=[0,\infty)$ endowed with the usual complex partial *b*-metric $\varsigma_{cb}: X\times X \to[0,\infty)$ defined by $\varsigma(\lambda,\mu)=[\max\{\lambda,\mu\}]^2(1+i)$. The complex partial b-metric space (X,ς_{cb}) is complete because (X,ς_{cb}^t) is complete with coefficient $s=1$. Indeed, for any $\lambda,\mu,\kappa\in X$,

$$\begin{aligned}\varsigma_{cb}^t &= 2\varsigma_{cb}(\lambda,\kappa)-\varsigma_{cb}(\lambda,\lambda)-\varsigma_{cb}(\kappa,\kappa)\\ &= 2[\max\{\lambda,\mu\}]^2(1+i)-(\lambda+i\lambda)-(\mu+i\mu)\\ &= |\lambda-\mu|^2+i\ |\lambda-\mu|^2\end{aligned}$$

Thus, (X,ς_{cb}) is the Euclidean complex metric space which is complete. Consider the mapping $\xi: X\times X\to X$ defined by $\xi(\lambda,\mu)=\dfrac{\lambda+\mu}{24}$. For any $\lambda,\mu,\kappa,\nu\in X$ we have

$$\begin{aligned}\varsigma_{cb}(\xi(\lambda,\mu),\xi(u,v)) &= \frac{1}{24}[\max\{\lambda+\mu\ ,\ u+v\}]^2(1+i)\\ &\le\frac{1}{24}[\max\{\lambda,u\}+\max\{\mu,v\}]^2(1+i)\\ &=\frac{1}{24}[\varsigma_{cb}(\lambda,u)+\varsigma_{cb}(\mu,v)]\end{aligned}$$

which is the contractive condition (9) for $\alpha = \frac{1}{12}$. Therefore, by Corollary 3.2, ξ has a unique coupled fixed point, which is $(0,0)$. Note that if the mapping $\xi : X \times X \to X$ is given by $\xi(\lambda,\mu) = \frac{\lambda+\mu}{2}$, then ξ satisfies the contractive condition (9) for $\alpha = 1$ that, is,

$$\varsigma_{cb}(\xi(\lambda,\mu),\xi(u,v)) = \frac{1}{2}\left[\max\{\lambda+\mu \ , \ u+v\}\right]^2 (1+i)$$

$$\leq \frac{1}{2}\left[\max\{\lambda,u\} + \max\{\mu,v\}\right]^2 (1+i)$$

$$= \frac{1}{2}\left[\varsigma_{cb}(\lambda,u) + \varsigma_{cb}(\mu,v)\right]$$

In this case, $(0,0)$ and $(1,1)$ are both coupled fixed points of ξ, and hence the coupled fixed point of ξ is not unique. This shows that the condition $\alpha < 1$ in Corollary 3.2, and hence $\alpha + \beta < 1$ in Theorem 2.1 cannot be omitted in the statement of the aforesaid results.

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