HARMONIC ANALYSIS OF EQUATION OF CENTRE IN *SĀRIṆĪS*

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Abstract: In astronomical tables, namely in *Makaranda sāriṇī* the equation of the centre (*mandaphala*) of each planet attains its maximum value for planet's *manda* anomaly beyond 90º. Whereas in almost all other Indian texts it is given that *mandaphala* attains its maximum value at 90º. This unusual behavior of *mandaphala* is analyzed by harmonic analysis and same is applied for the *mandaphala* of Mercury in *Karaṇakutūhala sāriṇī* because it attains maximum value for the *manda* anomaly at 88º.

In the present paper, we analyze mathematically the peripheries of planets and prescribe the new *manda* peripheries in the case of each planet so as to match with modern computational values.
Key words: Anomaly, apogee, equation of centre, equation of conjunction, Indian astronomy, longitude and periphery.

1. Introduction

In Indian astronomy, the heavenly bodies were assumed to move around the Sun in a circular orbit with uniform angular velocities. But by observation it was found that these motions were not uniform, hence some corrections were devised to obtain the true positions of planet.

The equation of the centre (*mandaphala*) and the equation of conjunction (*śīghraphala*) are the two important corrections applied to five planets (Mars, Mercury, Jupiter, Venus and Saturn) to obtain their true longitude. Of these the first corresponds to finding the true heliocentric longitude of the planet moving in an elliptic orbit as in modern astronomy and depends upon the anomalistic revolution. The second corresponds to converting the heliocentric longitude into geocentric longitude and depends on the synodic revolution.

In Indian astronomy, many astronomical tables are found based on different schools among them *Makaranda sāriṇī*(*MKS*) based on *Sūryasiddhānta*(*SS*)and *Karaṇakutūhala sāriṇī* (*KKS*) based on *Karaṇakutūhala* (*KK*) stands unique because the equation of centre of each planet attains its maximum value for planet's *manda* anomaly (*mk*) beyond 90º*,* whereas in almost all other Indian texts it is given that *mandaphala* attains its maximum value at *mk* = 90º.

This unusual behavior of equation of centre in *Makaranda sāriṇī,* is discussed by harmonic analysis and it is applied for the equation of centre of Mercury in *Karaṇakutūhala sāriṇī.* In *Karaṇakutūhala sāriṇī,* which attains its maximum value for *manda* anomaly = 88º.

2. Rationale for equation of centre (*mandaphala*):

In Indian classical texts, the equation of centre (*mandaphala*) of the planets is given by $\sin(MP) = \frac{p}{R} \times \sin(mk)$ -------------- (1)

where *MP* is *mandaphala* and *mk* is the *mandakendra* (*manda* anomaly). The Variable periphery (*paridhi*) of the *manda* epicycles is listed in table 2 as per *Sūryasiddhānta.*

$$p = p_e - (p_e - p_o)|\sin(mk)| \quad \text{-------------- (2)}$$

Let mk_1 be the *manda* anomaly of a planet corresponding to *manda* periphery (p_1) then the above equation (2) and (1) becomes

$$p_1 = p_e - (p_e - p_o)|\sin(mk_1)| \quad \text{------------- (3)}$$

$$\sin(MP_1) = \frac{p1}{R} \times \sin(mk_1) \quad \text{------------- (4)}$$

Now, the *mandakendra* (mk_1) be revised by adding half of the *mandaphala* obtained from (4), Let this revised *mandakendra* be mk_2 and the corresponding *mandaphala* be MP_2.

$$\sin(MP_2) = \frac{p1}{R} \times \sin(mk_2), \text{ where } mk_2 = mk_1 - \frac{1}{2}(MP_1) \quad \text{------------- (5)}$$

As an example, Let us verify the values of *mandaphala* obtained by using the peripheries given in the original text *Sūryasiddhānta* with the values given in *Makaranda sāriṇī* and the revised values obtained by using the formula (5) for the superior planet Mars and interior planet Venus.

Table 1: *Mandaphala* of Mars according to *SS, MKS* and the revised for *mk* varying from 0º to180º

Mandakendra *mk*	*Paridhi* *p*	*MP*(*SS*) in arc- minutes	*MP*(*MKS*) in arc- minutes	*MP*(Revised) in arc- minutes
0 º	75	0	0	0
15 º	74.224	183.53	165	165.3442
30 º	73.5	351.55	320	320.4608
45 º	72.879	493.80	458	457.8608
60 º	72. 402	601.83	570	569.9568
75 º	72.102	669.28	649	649.2464
90 º	72	692.22	689	688.8117
105 º	72.102	669.28	683	683.1411
120 º	72.401	601.83	629	629.1447
135 º	72.879	493.80	527	527.1520
150 º	73.5	351.55	381	381.6481
165 º	72.224	183.53	202	202.5605
180 º	75	0	0	0

In table 1, the revised values of *mandaphala* obtained by using the formula (5) for a planet Mars coincide with the values of *Makaranda sāriṇī* correct to an integer. Similarly, the revised values coincide with the values of *MKS* for the other two superior planets Jupiter and Saturn. Therefore the suggested algorithm (5) holds good in the case of superior planets. In order to study the further variation of the *mandaphala* close to the critical value of *mk*, its values for every degree has to be considered in a close interval of *mk* as given in *MKS.*

Table 2: *Mandaphala* of Venus according to *SS, MKS* and the revised for *mk* varying from 0º to180º

Mandakendra *mk*	*Paridhi* *p*	*MP*(*SS*) in arc- minutes	*MP*(*MKS*) in arc- minutes	*MP*(Revised) in arc- minutes
0 º	12	0	0	0
15 º	11.741	29.02	29	28.5717
30 º	11.5	54.91	54	54.1820
45 º	11.293	76.26	75	75.4621
60 º	11.134	92.09	92	91.4233
75 º	11.034	101.79	101	101.4133
90 º	11	105.06	105	105.0475
105 º	11.034	101.79	102	102.1505
120 º	11.134	92.09	93	92.7370
135 º	11.293	76.26	77	77.0481
150 º	11.5	54.91	56	55.6353
165 º	11.741	29.02	29	29.4658
180 º	12	0	0	0

In table 2, the revised values of *mandaphala* obtained by using the formula (5) for a planet Venus coincide with the values of *Makaranda sāriṇī* correct to an integer. Similarly, the revised values coincide with the values of *MKS* for other interior planet Mercury. Hence the suggested algorithm (5) holds good even in the case of interior planets.

As discussed earlier, now there is a necessity to verify the *mandaphala* for every degree of *mk* near the critical point for the purpose of further study in this area. Since the eccentricity of

Mercury's orbit is large, when compared with other four planets. The harmonic analysis of equation of centre is applied to *MP* of Mercury. Before analyzing the equation of centre by harmonic analysis, the values of equation of centre (*mandaphala*) of Mercury for an interval of every 15º between 0º to 180º and near the critical *mandakendra* varying for every degree from 88º to 95º is listed in tables 3 and 4. In *MKS* the *mandaphala* attains its maximum at *mk* = 92º and *mk* = 93º and the maximum *MP* = 4º28′ = 268′

Table 3: *Mandaphala* of Mercury according to *SS, MKS* and the revised for *mk* varying from 0º to180º

Mandakendra *mk*	*Paridhi* *p*	*MP*(*SS*) in arc- minutes	*MP*(*MKS*) in arc- minutes	*MP*(Revised) in arc- minutes
0 º	30	0	0	0
15 º	29.48	72.87	70	70.034
30 º	29	138.50	134	133.801
45 º	28.586	193.12	188	187.882
60 º	28.268	233.96	230	229.501
75 º	28.068	259.14	257	256.528
90 º	28	267.65	267	267.462
105 º	28.068	259.14	261	261.411
120 º	28.268	233.96	237	238.143
135 º	28.586	193.12	197	198.205
150 º	29	138.50	143	143.138
165 º	29.48	72.87	76	75.699
180 º	30	0	0	0

Table 4: *Mandaphala* of Mercury according to *SS, MKS* and the revised formula for *mk* varying from 88º to 97º

Mandakendra *mk*	*MP*(*SS*) in arc- minutes	*MP*(*MKS*) in arc- minutes	*MP*(*Revised*) in arc- minutes
88º	267.4989	267	266.9724
89º	267.6127	267	267.2549
90º	267.6506	267	267.4619
91º	267.6127	267	267.5932
92º	267.4989	268	267.6486
93º	267.3093	268	267.6280
94º	267.0439	267	267.5311
95º	266.7027	267	267.3580
96º	266.2858	267	267.1085
97º	265.0149	267	266.7824

From the above table 4, the equation of centre of Mercury attains its maximum at *mk* = 90º according to SS, at *mk* = 92º and 93º according to *MKS*, at *mk* = 92º according to revised formula, but if the values are corrected to an integer then it is at *mk* = 92º and 93º. In *MKS*, the critical *mk* is given over a range rather than at a single point, this is because the *mandaphala* is given in terms of degrees (*amśa*) and arc-minutes (*kalās*) but not in arc-seconds (*vikalā*)

3. Equation of centre (*mandaphala*) in modern astronomy:

The corresponding modern expression for the equation of the centre is

$$E = (2e - \tfrac{1}{4}e^3)\ \sin(m) + (\tfrac{5}{4}e^2 - \tfrac{11}{24}e^4)\sin(2m) + (\tfrac{13}{12}e^3 - \tfrac{43}{64}e^5)\sin(3m) + \tfrac{103}{96}e^4 \sin(4m) + \tfrac{1097}{960}e^5 \sin(5m) \qquad \text{--------------} \quad (6)$$

Here 'e' is the eccentricity of the planet's orbit, *E* is the equation of centre and '*m*' is the planet's anomaly from perigee, In Indian astronomical texts the anomaly is measured from apogee (*mandocca*). Therefore $m = 180^{o} - mk$, where $mk = mandocca -$ mean planet.

For Mercury, considering the eccentricity e = 0.20565 and using the formula (6), the critical *mk* and maximum *MP* was verified for the values between 88º to 97º and found that the *MP* did not reach maximum in that range but increasing slowly. So further, the range of *mk* increased from 97º to 110º , surprisingly the *MP* reached its maximum value 1487.4204 arc minutes at *mk* =104.7.

Considering the above drastic change in the critical *mk* and maximum *MP* of Mercury by modern expression, it is verified for other planets by taking their eccentricities and *mk* between 91º to 98º and found that all other planets viz, Mars, Jupiter, Venus and Saturn have their critical *mk* > 90º as listed in table 5.

Table 5 : Critical *mk* & maximum *MP* of planets according to modern expression

Planets	e	Critical *mk* in degrees	Maximum *MP* in arc-minutes
Mars	0.09349	96.7	673.80134
Mercury	0.20565	104.7	1487.4204
Jupiter	0.04904	93.5	353.1853
Venus	0.00678	90.5	48.8162
Saturn	0.06172	94.4	444.5771

Therefore the critical values given by the author of *Makaranda sāriṇī* can't be ignored now, because according to modern expression also the equation of centre MP reaches its maximum beyond 90º. Considering the rationale (5) obtained for *MKS*, the maximum *MP* of all planets are obtained for the exact critical value, and the same are compared with those obtained by modern expression.

Table 6: Critical *mk* & maximum *MP* according to *MKS* and Modern expression

Planets	***MKS***		**Modern expression**	
	Critical *mk* in degrees	Maximum *MP* in arc-minutes	Critical *mk* in degrees	Maximum *MP* in arc-minutes
Mars	95.7	692.21717	96.7	673.80134
Mercury	92.2	267.65059	104.7	1487.4204
Jupiter	92.5	305.98123	93.5	353.1853
Venus	90.9	105.05861	90.5	48.8162
Saturn	93.8	459.7353	94.4	444.5771

From table 6, the critical points according to *MKS* do not differ from the corresponding modern values by not more than 1º except for the planet Mercury. This exception may be due to its large eccentricity. The maximum values of *MP* according to *MKS* are close to modern values in the case of superior planets but differ for the interior planets due to their large and small eccentricities of Mercury and Venus respectively. This shows that classical Indian astronomers did not adequately account for the eccentricities in the case of interior planets. The *manda* periphery of both interior planets has to be revised to match with modern values. According to SS the *manda* periphery of Mercury varies from 28º to 30º, this should vary from 109º to 151º.2 to get modern value of *MP*= 24º 47′. The *manda* periphery of Venus varies from 11º to 12º, this should vary from 4º to 5º.1 to get modern value of *MP*= 0º 48′ 47′′.

4. Harmonic analysis of equation of centre (*mandaphala*)

The equation of centre (*mandaphala*) *MP* of planets represents sinusoidal curve in the interval 0º to 180º, which is periodic, hence it can be subjected to Harmonic analysis. According to Fourier series expansion, the equation of centre (*mandaphala*) *MP* is expressed as

$$MP = b_1 \sin(mk) + b_2 \sin(2mk) + b_3 \sin(3mk) + b_4 \sin(4mk) + b_5 \sin(5mk) + \ldots \quad (7)$$

where $b_1, b_2, b_3, b_4, \ldots$ are Fourier coefficients to be determined by using $b_k = \frac{2}{n}\sum y \sin(kx)$, n= number of divisions of the interval.

In this expansion each term on the RHS is called harmonics and hence the analysis is called harmonic analysis. By considering different number of harmonics, the critical *mk* is obtained. For simplification let $MP = y$ and $mk = x$ then above (7) becomes

$$y = b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + b_4 \sin(4x) + b_5 \sin(5x) + \quad (8)$$

Suppose $y = b_1 \sin(x)$ only one harmonic is considered then for any function to attain its maxima or minima its first order derivative must be equal to zero, which gives critical point. Now, $\frac{dy}{dx} = 0 \Rightarrow$ $b_1 \cos(x) = 0$

since $b_1 \neq 0$, therefore $\cos(x) = 0$, which gives $x = 90º$, that is $mk = 0$ which is the known trivial solution to critical *mandakendra.*

By considering two harmonics,

$$y = b_1 \sin(x) + b_2 \sin(2x)$$

$$\frac{dy}{dx} = b_1 \cos(x) + 2b_2 \cos(2x)$$

For critical point $\frac{dy}{dx} = 0$

$$b_1 \cos(x) + 2b_2 \cos(2x) = 0$$

$$b_1 \cos(x) + 2b_2 \,[2\cos^2(x) - 1] = 0$$

$$4 b_2 \cos^2(x) + b_1 \cos(x) - 2 b_2 = 0$$

which is a quadratic equation yields two solutions depending on the coefficients $b_1 = 692.66$; $b_2 = -34.74$, $\cos(x) = -0.049659$ or $+5.034259$, since $-1 < \cos(x) < +1$, by discarding the second value the first value gives $x = 92º.846$. So the improved $mk = 92º.846$.

By considering three harmonics,

$$y = b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x)$$

For critical point $\frac{dy}{dx} = 0$

$$b_1 \cos(x) + 2b_2 \cos(2x) + 3 b_3 \cos(3x) = 0$$

$$b_1 \cos(x) + 2b_2 \,[2\cos^2(x) - 1] + 3b_3 \,[4\cos^3(x) - 3\cos(x)] = 0$$

$12b_3 \cos^3(x) + 4 b_2 \cos^2(x) + (b_1 - 9b_3)\cos(x) - 2 b_2 = 0$ on solving this cubic equation by Newton –Raphson method, the critical $mk = x$ can be determined. In this way the number of harmonics can be increased to get accurate value of critical *mk* and the corresponding maximum equation of centre *MP*. Similarly the expansion can be extended to more number of harmonics to obtain the accuracy in the critical value.

For the values of *MKS,* by considering the five harmonics for the planet Mars the coefficients b_1, b_2 , b_3, b_4 and b_5 are found as shown in table 7

$$y = b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + b_4 \sin(4x) + b_5 \sin(5x)$$

For critical point $\frac{dy}{dx} = 0$

$$b_1 \cos(x) + 2b_2 \cos(2x) + 3 b_3 \cos(3x) + 4 b_4 \cos(4x) + 5b_5 \cos(5x) = 0$$

$\Rightarrow$ 80 $b_5 \cos^5(x) + 32b_4 \cos^4(x) + (12b_3 - 100b_5)\cos^3(x) + (4b_2 - 32b_4)\cos^2(x) + (b_1 - 9b_3 + 25b_5)\cos(x) + (4b_4 - 2b_2) = 0$ -------------- (9)

Table 7: Harmonic analysis of mandaphala of Mars based on *MKS* values

mk x	*MP* (*MKS*) *y*	*y* sin *x*	*y*sin(2*x*)	*y*sin(3*x*)	*y*sin(4*x*)	*y*sin(5*x*)
0º	0	0	0	0	0	0
15º	165	42.705	82.5	116.672	142.894	159.378
30º	320	160	277.128	320	277.128	160
45º	458	309.713	483	309.712	0	-309.713
60º	570	493.634	493.635	0	- 493.634	-493.634
75º	649	626.885	324.5	- 458.912	- 562.05	167.973
90º	689	689	0	-689	0	689
105º	683	659.727	-341.5	- 482.953	591.495	176.773
120º	629	544.730	-544.730	0	544.73	-544.730
135º	527	372.645	-527	372.645	0	-372.645
150º	381	160.5	-277.994	321	-277.994	160.5
165º	202	52.281	-101	142.835	-174.937	195.117
SUM	**--**	**4111.82**	**-131.461.**	**-48.001**	**47.632**	**-11.981**

$b_1 = \frac{2}{12} \sum y \sin(x) = 685.303$; $b_2 = \frac{2}{12} \sum y \sin(2x) = -21.9$

$b_3 = \frac{2}{12} \sum y \sin(3x) = -8.0002$; $b_4 = \frac{2}{12} \sum y \sin(4x) = 7.9387$

$b_5 = \frac{2}{12} \sum y \sin(5x) = -1.9968$

These coefficients gives critical *mk* = 95.612 and maximum *MP* = 691.932 arc- minutes by solving the above equation (9) which is close to the value obtained under 2 (listed in table 6). Similarly this analysis can be applied to other planets.

5. Equation of centre (*mandaphala*) according to *Karaṇa kutūhala sārinī* :

In *Siddhānta śiromaṇi* of Bhāskara II, the peripheries *'p'* of *manda* epicycles are fixed, the same is followed in his *Karaṇa kutūhala* and as also as in *Karaṇa kutūhala sārinī.* According to that

$$MP = \frac{p}{R} \sin(mk) \qquad \text{--------- (10)}$$

sine of an angle attains maximum value for an angle = 90º, hence the *MP* attains maximum for *mk* = 90º but in *Karaṇa kutūhala sārinī* the *mandaphala* of Mercury attains its maximum at 88º. According to modern expression with eccentricity, the equation of centre *MP* of Mercury attains maximum value at *mk* =104.7º. Therefore there is a necessity to analyze the algorithm adopted by the author of *Karaṇa kutūhala sārinī.* Let the revised *mandaphala* of planets be given by the formula (5) with fixed periphery p =38º. If *mk* =90 º then MP_1 *and* MP_2 are given by

$MP_1 = [\frac{p}{R}]$ and

$MP_2 = [\frac{p}{R} \times \sin(mk_1)]$,

where $mk_1 = mk - \frac{1}{2}(MP_1) = 90^\circ - \frac{1}{2}(MP_1)$ -------------- (11)

Table 8, gives the list of maximum *MP* according to both *Karaṇakutūhala and Karaṇa kutūhala sārinī*

Table 8: Maximum *MP* according to *Karaṇakutūhala and Karaṇa kutūhala sārinī*

Planets	Manda periphery *p*	Maximum *MP* acc to the text *KK* at *mk* =90º	Maximum *MP* acc to *KKS* *at mk* =90º except Mercury	Maximum *MP* acc to Formula (10)
Mars	70º	672′ 54″	11° 12' 53"	11° 05' 18"
Mercury	38º	362′ 10″	6° 25' 25"	6° 02' 22"
Jupiter	33º	315′ 43″	5° 15' 47"	5° 14' 48"
Venus	11º	110′ 00″	1° 31' 50"	1° 45' 18"
Saturn	50º	458′ 33″	7° 38' 35"	7° 56' 19"

In spite of revising the formula for *mandaphala* the values are not matching it means that the author of *KKS* has not only revised the formula, even he has revised the peripheries.

6. Harmonic analysis of equation of centre (*mandaphala*) of Mercury according to *KKS*

Let the harmonic analysis be applied to the values of equation of centre (*mandaphala*) of Mercury to verify the required critical value, by considering 5 harmonics as given in table 9. The equation is

$MP = b_1 \sin(mk) + b_2 \sin(2mk) + b_3 \sin(3mk) + b_4 \sin(4mk) + b_5 \sin(5mk)$ -----------(12)

Table 9: Harmonic analysis of mandaphala of Mars based on *MKS* values

mk x	*MP* (*MKS*) *y*	*y* sin *x*	ysin(2*x*)	*y*sin(3*x*)	*y*sin(4*x*)	*y*sin(5*x*)
0º	0	0	0	0	0	0
15º	1º40′00″	25.882	50	70.711	86.6025	96.592
30º	3º01′49″	90.91	157.46	181.82	157.46	90.91
45º	4º16′03″	181.055	256.05	181.055	0	-181.055
60º	5º15′52″	273.551	273.551	0	-273.551	-273.551
75º	5º50′00″	338.074	175	-247.487	-303.109	90.587
90º	6º03′38″	363.63	0	-363.63	0	363.63
105º	5º50′00″	338.074	- 175	-247.487	303.109	90.587
120º	5º15′52″	273.551	-273.551	0	273.551	-273.551
135º	4º16′03″	181.055	-256.05	181.055	0	-181.055
150º	3º01′49″	90.91	-157.46	181.82	-157.46	90.91
165º	1º40′00″	25.882	-50	70.711	-86.602	96.592
SUM	**--**	**2182.572**	**0**	**8.568**	**0**	**10.596**

$b_1 = \frac{2}{12}\sum \square \sin(\square) = 363.762$; $b_2 = \frac{2}{12}\sum \square \sin(2\square) = 0$

$b_3 = \frac{2}{12}\sum \square \sin(3\square) = 1.428$; $b_4 = \frac{2}{12}\sum \square \sin(4\square) = 0$

$b_5 = \frac{2}{12}\sum \square \sin(5\square) = 1.766$

The equation (12) becomes

$MP = b_1 \sin(mk) + b_3 \sin(3mk) + b_5 \sin(5mk)$

Now $\frac{\square\square\square}{\square\square\square} = 0$ gives $b_1 \cos(mk) + 3 b_3 \cos(3mk) + 5b_5 \cos(5mk) = 0$

$\Rightarrow 80b_5 \cos^5(mk) + (12b_3 - 100b_5)\cos^3(mk) + (b_1 - 9b_3 + 25b_5)\cos(mk) = 0$

$\Rightarrow \cos(mk) = 0$ and other roots are imaginary

$\Rightarrow mk = 90^{\circ}$

This is again a contradictory to *KKS*'s maximum *mandaphala* at critical point *mk* = 88º. Hence the author of *KKS* would have revised the *manda* periphery along with critical value of *mandakendra*.

7. Conclusion:

The eccentricity of Mercury is quite large and that of Venus is quite small compared to other planets. It seems that classical Indian astronomers did not adequately account for the eccentricities in the case of interior planets. The *manda* periphery of both interior planets has to be revised to match with modern values. According to *SS* the *manda* periphery of Mercury varies from 28º to 30º, this should vary from 109º to 151º.2 to get modern value of *MP*= 24º 47′. The *manda* periphery of Venus varies from 11º to 12º, this should vary from 4º to 5º.1 to get modern value of *MP*= 0º 48′ 47″. We prescribe these new *manda* peripheries in the case of interior planets.

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