ISSN: 2278-4632
Vol-12 Issue-10 No.02 October 2022

Fuzzy Transportation ProblemUsing Ranking Approach

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ABSTRACT

The fuzzy set theory has been applied in many allied fields such as operation research, management science and control theory etc. The fuzzy numbers and fuzzy values are widely used in engineering application. In this paper, we propose a new ranking method for solving fully fuzzy transportation problem (FFTP). In ranking method, the given FFTP is converted into a crisp transportation problem (CTP) and solved by using Yager's ranking technique and the optimal solution to the given FFTP is obtained and then compared between our purposed method and the existing method. Numerical example is also provided to demonstrate the effectiveness and accuracy of our proposed method.

Keywords: *Fuzzy Transportation Problem, Symmetric Triangular Fuzzy Numbers, Ranking, Optimal Solution.*

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1. INTRODUCTION

Among linear programming problem, the transportation problem is very popular and special case of it, in which a commodity is to be transported from various sources of supply to various destination of demand in such a way that the total transportation cost is a minimum. But in practice, supply, demand and unit life transportation cost are uncertain due to several factors. These imprecise data may be better represented by fuzzy numbers. The transportation problem, in which the transportation costs, supply and demand quantities are represented in terms of fuzzy numbers, is called a fuzzy transportation problem.

To solve this type of fuzzy transportation problem many researchers Harrera and Verdegay, Ishibuchi and Tanka, Pandian and Natarajan [4,5,8] have proposed fuzzy and interval programming techniques for solving the transportation problem. Zadeh [13] introduced the concept of fuzzy numbers. Buckley and Feuring [1] proposed a method to find the solution for a fully fuzzified linear programming problem by changing the objective function into a multi objective linear programming problem. Liu and Kao[7] ,Chanas et al.[2], Chanas and Kuchta [3] proposed a method for solving fuzzy transportation problem. Samual and Gani[11] used Arshamrhan's algorithm to solve a fuzzy transportation problem. Pandian and Natarajan [8] proposed a fuzzy zero point method for finding a fuzzy optimal solution for fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. Rani et al. [10] proposed a new method for finding an optimal solution for fully fuzzy unbalanced transportation problem.

In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal. Thus, some fuzzy numbers are not directly comparable. Comparing between two or multi fuzzy numbers and ranking such numbers is one of the important subjects, and how to set the rank of fuzzy numbers has been one of the main problems.

Here we propose a method in which ranking of fuzzy triangular numbers used by Yager[12] ranking method to transform the fuzzy transportation problem to a crisp one so that the conventional solution method may be applied to solve the transportation. This method is very easy to understand and to apply. At the end, the optimal solution of problem can be obtained in a fuzzy number or a crisp form.

2. DEFINITIONS & NOTATIONS

In this section, some necessary definitions and notions of fuzzy set theory are reviewed.

Membership Function & Fuzzy Set

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function μ_A such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_A : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A.

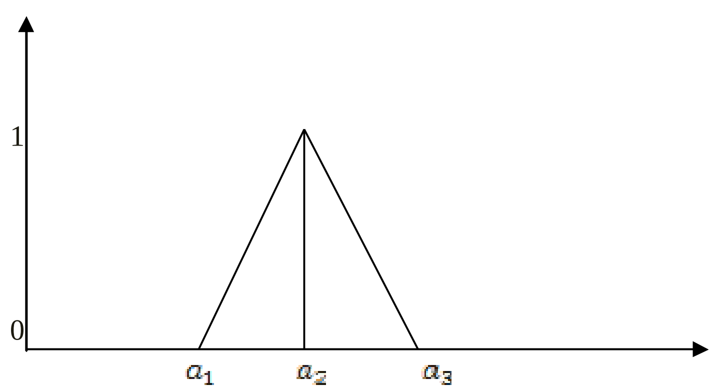
The function μ_A is called the membership function and the set $\tilde{A} = \{(x, \mu_A(x)); x \in X\}$ defined by $\mu_A(x)$ for each $x \in X$ is called a fuzzy set.

Triangular Fuzzy Number

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

The geometric representation of triangular fuzzy number is shown in figure. Since, the shape of the triangular fuzzy number *A* is usually in triangle it is called so.



ISSN: 2278-4632
Vol-12 Issue-10 No.02 October 2022

The triangular fuzzy number is based on three-value judgment: The minimum possible value a_1, the most possible value a_2 and the maximum possible value a_3.

- *A* triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be *non-negative fuzzy number* iff $a_1 \geq o$.

- In a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ if $a_2 = a_3$ than it is called *symmetric triangular fuzzy number*. It is denoted by $\tilde{A} = (a_1, a_2, a_2)$

α-cut

Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, the α-cut, α_A is the crisp set $\alpha_A = \{x \,/\, A(x) \geq \alpha\}$.

- Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, the α-cut, α_{A^+} is the crisp set $\alpha_{A^+} = \{x \,/\, A(x) > \alpha\}$.

Some Properties

Let (a_1, a_2, a_3) *and* (b_1, b_2, b_3) be two triangular fuzzy numbers, then some mathematical notation/operations are as follows

(i) $(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
(ii) $(a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
(iii) $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ $for\ k \geq 0$
(iv) $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$ $for\ k < 0$
(v) $(a_1, a_2, a_3) \times (b_1, b_2, b_3) = (a_1 b_1, a_2 b_2, a_3 b_3)$ $for\ a_1 > 0$
$= (a_1 b_3, a_2 b_2, a_3 b_3)$ $for\ a_1 < 0, a_3 \geq 0$
$= (a_1 b_3, a_2 b_2, a_3 b_1)$ $for\ a_3 > 0$

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be in F(R), then some relational notation/operations are as follows

(i) $\tilde{A} < \tilde{B}$ iff $R(\tilde{A}) < R(\tilde{B})$
(ii) $\tilde{A} > \tilde{B}$ iff $R(\tilde{A}) > R(\tilde{B})$
(iii) $\tilde{A} = \tilde{B}$ iff $R(\tilde{A}) = R(\tilde{B})$
(iv) $\tilde{A} - \tilde{B} = 0$ iff $R(\tilde{A}) - R(\tilde{B}) = 0$

A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3) \in F(R)$ is said to be positive if $R(\tilde{A}) > 0$ and denoted by $\tilde{A} > 0$ and if $R(\tilde{A}) = 0$ then $\tilde{A} = 0$, if $R(\tilde{A}) = R(\tilde{B})$ then the triangular numbers $\tilde{A}$ and $\tilde{B}$ are said to be equivalent and is denoted by $(\tilde{A}) = (\tilde{B})$.

3. RANKING FUNCTION

A convenient method for comparing of the fuzzy numbers is by use of ranking function. A ranking function is a map from F(R) into the real line. Since there are many ranking function for comparing fuzzy numbers, here we have applied Yager's ranking technique satisfies compensation, linearity and additive properties provides results which are consistent with human intuition.

Given a convex fuzzy number $\tilde{a}$, then the Yager's ranking index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u)\,d\alpha$$

Where (a_α^l, a_α^u) is the α level cut of the fuzzy number $\tilde{a}$ and

$$a_\alpha^l = (a_2 - a_1)\alpha + a_1,$$
$$a_\alpha^u = -(a_3 - a_2)\alpha + a_3.$$

4. MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM

Mathematically, a fuzzy transportation problem (FTP) can be stated as:

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}\tilde{X}_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} \tilde{X}_{ij} \approx \tilde{a}_i, \quad i = 1,2,\dots\dots\dots m$$

$$\sum_{i=1}^{m} \tilde{X}_{ij} \approx \tilde{b}_j, \quad j = 1,2,\dots\dots\dots n$$

and $\tilde{X}_{ij} \geq 0$

Where,

m = total number of origins

n = total number of destinations

$\tilde{a}_i$= fuzzy availability of commodity at i^{th} origin

$\tilde{b}_j$= fuzzy commodity needed at the j^{th} destination

$\tilde{C}_{ij}$= fuzzy transportation cost of one unit of product from i^{th} origin to j^{th} destination

$\tilde{X}_{ij}$=fuzzy quantity transported from i^{th} origin to j^{th} destination

$$\tilde{a}_i = \left[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}\right],\ \tilde{b}_j = \left[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}\right],\ \tilde{C}_{ij} = \left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}\right],\ \tilde{X}_{ij} = \left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}\right]$$

The above fuzzy transportation problem is said to be balanced if $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$, otherwise it is called unbalanced.

ISSN: 2278-4632
Vol-12 Issue-10 No.02 October 2022

This tabular representation of the problem is as follows:

	1	....	N	Supply
1	$\tilde{c}_{11}$	....	$\tilde{c}_{1n}$	$\tilde{a}_1$
2	$\tilde{c}_{21}$	....	$\tilde{c}_{2n}$	$\tilde{a}_2$
. . . .	. . . .	. . . .	. . . .	. . . .
M	$\tilde{c}_{m1}$	....	$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	....	$\tilde{b}_n$	

5. ALGORITHM

This proposed method in algorithm form for finding the optimal basic feasible solution in a symmetric triangular fuzzy environment and step by step procedure is as follows:

Step1. From the given data, construct the transportation table whose cost matrix, supplies and demands are fuzzy symmetric triangular numbers.

Step 2. Calculate Yager's ranking index for each cell of transportation table.

Step 3. Replace symmetric triangular numbers by their respective ranking indices.

Step 4. Solve the resulting transportation problem by using existing Vogel's method to find the optimal solution.

The all step by step procedure is explained in next section of numerical example.

6. NUMERICAL EXAMPLE

Consider the following fuzzy transportation problem

	Destination				**Supply**
Source	(2,3,3)	(2,3,3)	(2,3,3)	(1,4,4)	(0,3,3)
	(4,9,9)	(4,8,8)	(2,5,5)	(1,4,4)	(2,13,13)
	(2,7,7)	(0,5,5)	(0,5,5)	(4,8,8)	(2,8,8)
Demand	(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Convert the given fuzzy problem into a crisp value problem by using the measure.

Y (2, 3, 3) = $\int_0^1 0.5(2 + \alpha + 3)\,d\alpha$

$= \int_0^1 0.5(5+\alpha)\,d\alpha$

= 2.75

Similarly,

Y (1, 4, 4) = 3.25 Y (2, 5, 5) = 4.25
Y (4, 9, 9) = 7.75 Y (4, 8, 8) = 7
Y (2, 7, 7) = 5.75 Y (0, 3, 3) = 2.25
Y (0, 5, 5) = 3.75 Y (2, 8, 8) = 6.5
Y (0, 9, 9) = 6.75 Y (2, 13, 13) = 10.2

ISSN: 2278-4632
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Now, the reduced transportation problem is as follows:

	Destination				**Supply**
Source	2.75	2.75	2.75	3.25	2.25
	7.75	7	4.25	3.25	10.25
	5.75	3.75	3.75	7	6.5
Demand	3.25	6.75	3.25	5.75	

Using Vogel's procedure we obtain the initial solution as

	Destination				**Supply**
Source	**2.25**(2.75)				2.25
	1(7.75)	**0.25**(7)	**3.25**(4.25)	**5.75**(3.25)	10.25
		6.5(3.75)			6.5
Demand	3.25	6.75	3.25	5.75	

Now using the allotment rules, the solution of the problem can be obtained in the form of symmetric triangular fuzzy numbers.

	Destination				**Supply**
Source	(0,3,3)				(0,3,3)
	(1,1,1)	(-2,1,1)	(1,4,4)	(2,7,7)	(2,13,13)
		(2,8,8)			(2,8,8)
Demand	(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Hence the crisp optimal solution is **72.5625** and the fuzzy optimal solution for the given transportation problem is

$$x_{11} = (0,3,3),\; x_{21} = (1,1,1), \quad x_{22} = (-2,1,1), \quad x_{23} = (1,4,4),$$
$$x_{24} = (2,7,7), \quad x_{32} = (2,8,8).$$

7. COMPARISON

In the proposed method we obtain the optimal solution for the above fuzzy transportation problem is 72.5625 and

$$x_{11} = (0,3,3), x_{21} = (1,1,1), x_{22} = (-2,1,1), x_{23} = (1,4,4),$$
$$x_{24} = (2,7,7), x_{32} = (2,8,8).$$

By Naresh kumar & Ghuru [7] method the optimal solution for the above fuzzy transportation problem is **84.33** and

$$x_{14} = (0,3,3), x_{21} = (1,4,4), x_{22} = (-2,1,1),$$
$$x_{23} = (1,4,4), x_{24} = (2,4,4,), x_{32} = (2,8,8).$$

ISSN: 2278-4632
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Method	Optimal solution
Kumar S.Naresh & Ghuru Method	84.33
Proposed method	72.5625

8. CONCLUSION

In this paper a new ranking method for solving fully fuzzy transportation problem (FFTP) is discussed and illustrated with suitable numerical example. We have also compared it with method given by Kumar and Ghuru [6]. In this paper the general fuzzy numbers and the decision variables are considered as symmetric triangular fuzzy numbers. The effectiveness of our proposed method is demonstrated by using the same example given by Kumar and Ghuru [6]. It is obvious from the results shown in the final table that by using proposed method the ranking value and the optimal value are less than existing method. The solution to the problem is given both as a fuzzy number and also as a ranked fuzzy number. The proposed method is very easy to understand and can be applied for fully fuzzy transportation problem occurring in real life situation.

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