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(M/M/C:C/FCFS) QUEUE WITH TWO-CLASS ARRIVALS, STATE DEPENDENT SERVICE, MULTI-SERVERS AND CUSTOMERS IMPATIENCE

Mahendra Varma Polakonda Research Scholar, Department of Engineering Mathematics, College of Engineering, Koneru Lakshmaiah Education Foundation, Vaddeswaram-522302, Andhra Pradesh, India.
V.N.Rama Devi Professor Department of Mathematics, GRIET, Hyderabad, Telangana, India.
Rajyalakshmi Kottapalli, Assistant Professor, Department of CSE, Koneru Lakshmaiah Education Foundation, Green fileds, Vaddeswaram, Guntur-522303, Andhra Pradesh, India.
e-mail: mahendravarma12@gmail.com; ramadevivn@gmail.com;
rajyalakshmi.kottapalli@gmail.com

Abstract

This paper deals with the finite Markovian queueing system with multi server facility, customer impatience and two types of arrivals with heterogeneous arrival rates along with state dependent service. Arrival and departures assumed to be Poisson and exponential distributions respectively under first-come-first-serve basis. Transient state probabilities as well as some performance indices have been calculated. We have presented the results of sensitivity analysis and observed the impact of various parameters on system's constants.

Key words: Two-class customers, Balking, Reneging and multi-server facility

I Introduction

Queueing theory is a part of Operations Research that is used for drawing optimum solutions to provide service with the given constraints. Queues help organisations to render service in an orderly manner. The analysis is about fixing a mathematical model with the given arrival and service rates. Recent times are witnessing great attention in queueing models due to their wide applications.

Haight [6] was the first to investigate the concept of customer impatience. He considered a balking model for M/M/1 queues in which an arrival would not balk at the longest queue length. Balking (refusing to join the queue immediately upon the arrival) and Reneging were two of the Queuing Problems detailed by Ancker and Gafarian [2]. Later many researchers [1,3,13,15,16] have made significant contribution in this aspect.

Many researchers have focussed on techniques that can reduce customer impatience. Concept of multi-servers is one among them. Y.Levi et.al[18] have studied and obtained performance measures for an M/M/s queue with vacation. Davis et.al [14] considered a multi-server queueing model in a priority queue and derived key inputs. Lot of literature [7,8,9,10,11] is available in this domain.

Customers arrive the queue in different ways based on the need of the service. Different arrival rates are also possible for the same kind of the service in real life. Studies are many in this area. An M/M/m

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queue with two classes of consumers and several vacations was examined by MingzhouXie et.al [12]. Studies [4,5,12,17] are significant in this domain.

In this paper, we present the transient analysis of M/M/C queue with two types of arrivals and state dependent service rates using R-K method. The work is presented as follows: Section 2 describes the model, section 3 is about Transient state model and respective probabilities, Section 4, is of presentation of some constants of the system through numerical results and sensitivity analysis. Section 5 presents final conclusions.

II About the Model

We analyzed a finite Markovian Queue having multi -channel facility with heterogenous arrivals, state dependent service and Customer Impatience in Transient mode with the following assumptions:

1. The capacity of the system is considered as c(finite).
2. The number of service channels is c.
3. The mean arrival rate of Type-I customers is λ_1.
4. The mean arrival rate of Type-II customers is λ_2.
5. The mean service rate of servers of Type-I customers is μ_1.
6. The mean service rate of servers of Type-II customers is μ_2.
7. The balking parameter is b.
8. $\pi_{x,y}^{(t)}$ denotes the probability that there are "x" customers of Type-I and "y" customers of Type-II at time point "t".

The following differential equations are formed to compute various probabilities:

$$\frac{d\pi_{0,0}^{(t)}}{dt} = -(\lambda_1+\lambda_2)b\pi_{0,0}^{(t)} + \mu_1\pi_{1,0}^{(t)} + \mu_2\pi_{0,1}^{(t)} \tag{1}$$

$$\frac{d\pi_{x,0}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1)\,\pi_{x,0}^{(t)} + \lambda_1 b\pi_{x-1,0}^{(t)} + (x+1)\mu_1\pi_{x+1,0}^{(t)} + \mu_2\pi_{x,1}^{(t)};\ 1 \le x \le c-1 \tag{2}$$

$$\frac{d\pi_{0,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + y\mu_2)\,\pi_{0,y}^{(t)} + \lambda_2 b\pi_{0,y-1}^{(t)} + (y+1)\,\mu_2\pi_{0,y+1}^{(t)} + \mu_1\pi_{1,y}^{(t)};\ 1 \le y \le c-1 \tag{3}$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1 + y\mu_2)\,\pi_{x,y}^{(t)} + \lambda_1 b\pi_{x-1,y}^{(t)} + \lambda_2 b\pi_{x,y-1}^{(t)} + (x+1)\,\mu_1\pi_{x+1,y}^{(t)} + (y+1)\,\mu_2\pi_{x,y+1}^{(t)};$$
$$x \text{ and } y \neq 0 \text{ and } x+y \le c-1 \tag{4}$$

$$\frac{d\pi_{c,0}^{(t)}}{dt} = -(c\mu_1)\,\pi_{c,0}^{(t)} + \lambda_1 b\pi_{c-1,0}^{(t)} \tag{5}$$

$$\frac{d\pi_{0,c}^{(t)}}{dt} = -(c\mu_2)\,\pi_{0,c}^{(t)} + \lambda_2\, b\pi_{0,c-1}^{(t)} \tag{6}$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -(x\mu_1 + y\mu_2)\pi_{x,y}^{(t)} + \lambda_1 b\pi_{x-1,y}^{(t)} + \lambda_2 b\pi_{x,y-1}^{(t)};\quad x \text{ and } y \neq 0 \text{ and } x+y = c \tag{7}$$

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III Performance Measures

Queueing measurements that are computed to forecast the system are

1. Expected length of system ($L_S^{(t)}$)
2. Mean waiting time ($W_S^{(t)}$)

IV Numerical Results

The numerical values for Mean length, waiting time and also the effect of various parameters are calculated by using MATLAB for Runge-Kutta for the given values of the parameters. These values are taken by verifying the traffic intensity condition. They are presented in Tables 1-5 and Figures 1-5.

The model parameters are considered as follows:

$$c = 10, \ \lambda_1 = .02, \lambda_2 = .03, \mu_1 = .04, \mu_2 = .05, b = 0.3$$

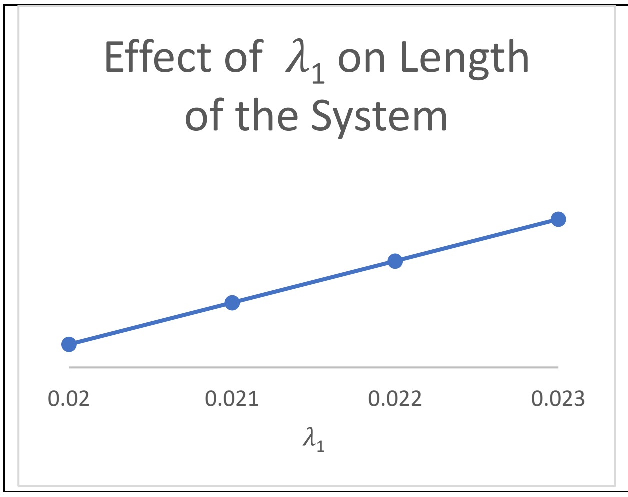
The time instances are taken as follows:

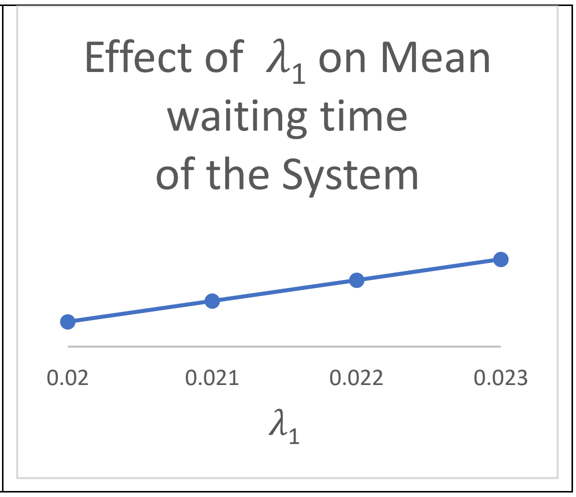
$$t_1 = 0.5, t_2 = 1.0, t_3 = 1.5, t_4 = 2.0$$

Table 1: Effect of λ_1

Parameter (λ_1)	t	t_1	t_2	t_3	t_4
0.02	$L_S^{(t)}$	0.007414415	0.014660288	0.021741492	0.028661813
	$W_S^{(t)}$	0.297748544	0.590990281	0.879723586	1.1639521
0.021	$L_S^{(t)}$	0.007562925	0.014954367	0.022178258	0.02923844
	$W_S^{(t)}$	0.297770446	0.591075581	0.879910483	1.164275712
0.022	$L_S^{(t)}$	0.007711435	0.015248446	0.022615024	0.029815068
	$W_S^{(t)}$	0.297792349	0.591160881	0.880097379	1.164599324
0.023	$L_S^{(t)}$	0.007859945	0.015542525	0.02305179	0.030391695
	$W_S^{(t)}$	0.297814252	0.591246181	0.880284276	1.164922938

Figure 1: Effect of λ_1





Inference: From the above table and figures, it is observed that the expected queue length as well as mean waiting times are raising with respect to a raise in Type-I arrival rate λ_1.

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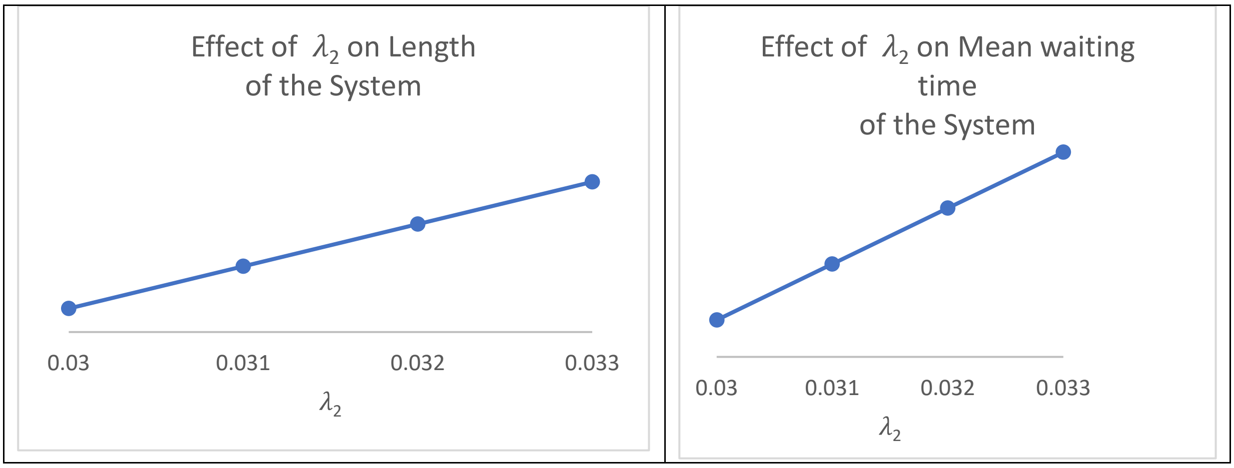
Table 2: Effect of λ_2

Parameter (λ_2)	t	t_1	t_2	t_3	t_4
0.03	$L_S^{(t)}$	0.007414415	0.014660288	0.021741492	0.028661813
	$W_S^{(t)}$	0.297748544	0.590990281	0.879723586	1.1639521
0.031	$L_S^{(t)}$	0.007562555	0.014952911	0.022175032	0.029232788
	$W_S^{(t)}$	0.297770479	0.591075833	0.879911303	1.164277591
0.032	$L_S^{(t)}$	0.007710696	0.015245535	0.022608571	0.029803764
	$W_S^{(t)}$	0.297792414	0.591161384	0.880099021	1.164603083
0.033	$L_S^{(t)}$	0.007858836	0.015538158	0.02304211	0.030374739
	$W_S^{(t)}$	0.29781435	0.591246936	0.880286738	1.164928576

Figure 2: Effect of λ_2



Inference: From the table and figures, it is observed that the expected queue length as well as mean waiting times are raising with respect to a raise in Type-II arrival rate λ_2.

Table 3: Effect of μ_1

Parameter (μ_1)	t	t_1	t_2	t_3	t_4
0.04	$L_S^{(t)}$	0.0074144148	0.0146602877	0.0217414924	0.0286618127
	$W_S^{(t)}$	0.2977485435	0.5909902807	0.8797235864	1.1639521003
0.041	$L_S^{(t)}$	0.0074136749	0.0146573675	0.0217350096	0.0286504415
	$W_S^{(t)}$	0.2977112728	0.5908421450	0.8793924775	1.1633674725
0.042	$L_S^{(t)}$	0.0074129352	0.0146544492	0.0217285333	0.0286390852
	$W_S^{(t)}$	0.2976740144	0.5906941077	0.8790616979	1.1627836182
0.043	$L_S^{(t)}$	0.0074121957	0.0146515328	0.0217220634	0.0286277440
	$W_S^{(t)}$	0.2976367684	0.5905461687	0.8787312471	1.1622005362

Figure 3: Effect of μ_1

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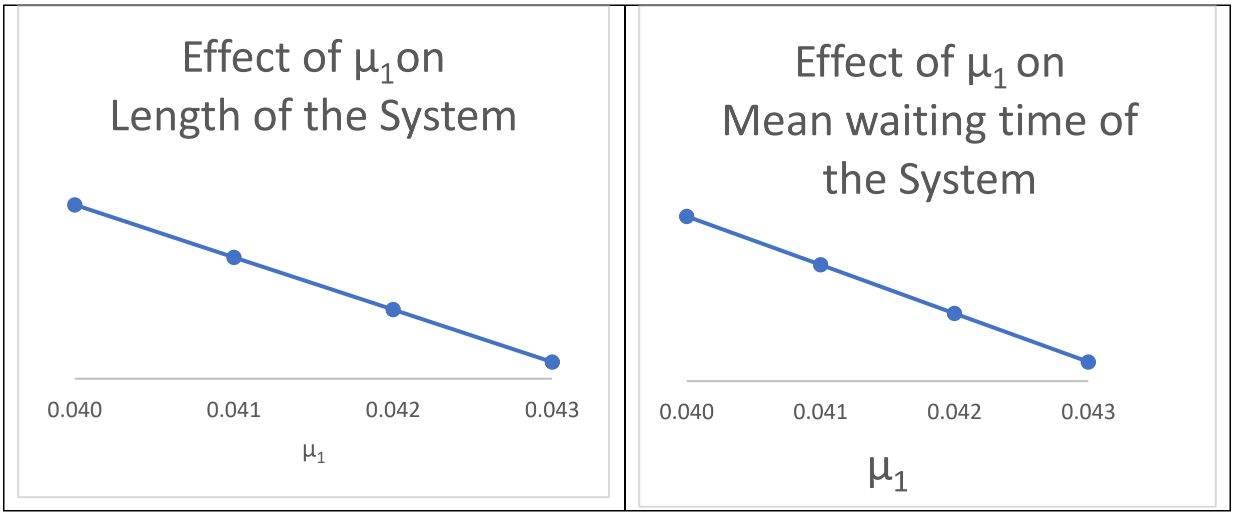
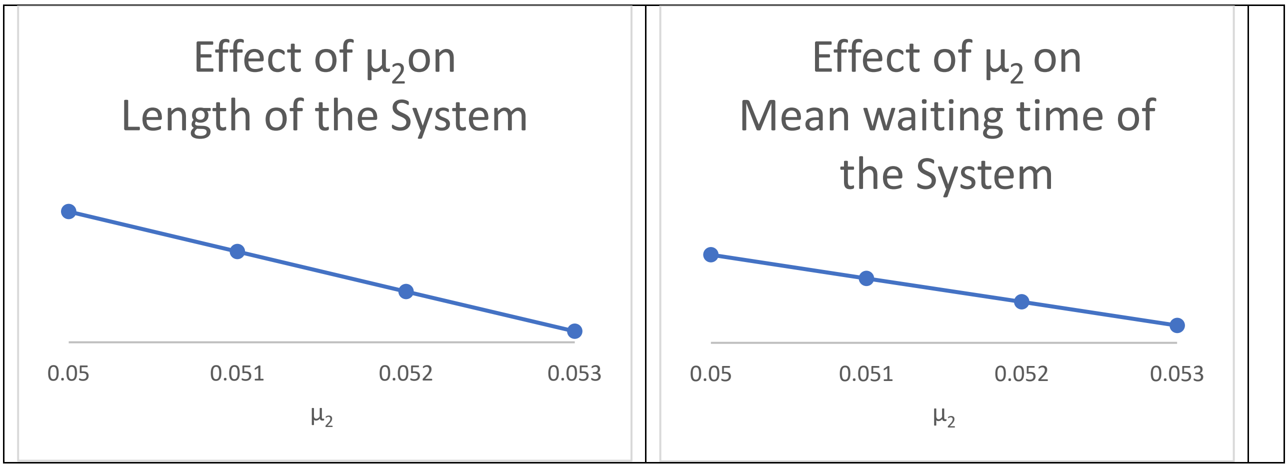


Inference: From the above table and figures, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to a decline in Type-I service rate μ_1.

Table 4: Effect of μ_2					
Parameter (μ_2)	t	t_1	t_2	t_3	t_4
0.035	$L_S^{(t)}$	0.007414415	0.014660288	0.021741492	0.028661813
	$W_S^{(t)}$	0.297748544	0.590990281	0.879723586	1.1639521
0.036	$L_S^{(t)}$	0.007413309	0.014655936	0.021731864	0.02864498
	$W_S^{(t)}$	0.297711396	0.590843115	0.879395702	1.163374999
0.037	$L_S^{(t)}$	0.007412203	0.014651588	0.021722246	0.02862817
	$W_S^{(t)}$	0.297674261	0.590696047	0.879068143	1.162798661
0.038	$L_S^{(t)}$	0.007411097	0.014647242	0.021712637	0.028611381
	$W_S^{(t)}$	0.297637138	0.590549077	0.878740909	1.162223083

Figure 4: Effect of μ_2



Inference: From the above table and figures, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to a decline in Type-I service rate μ_2.

Table 5 Effect of *b*

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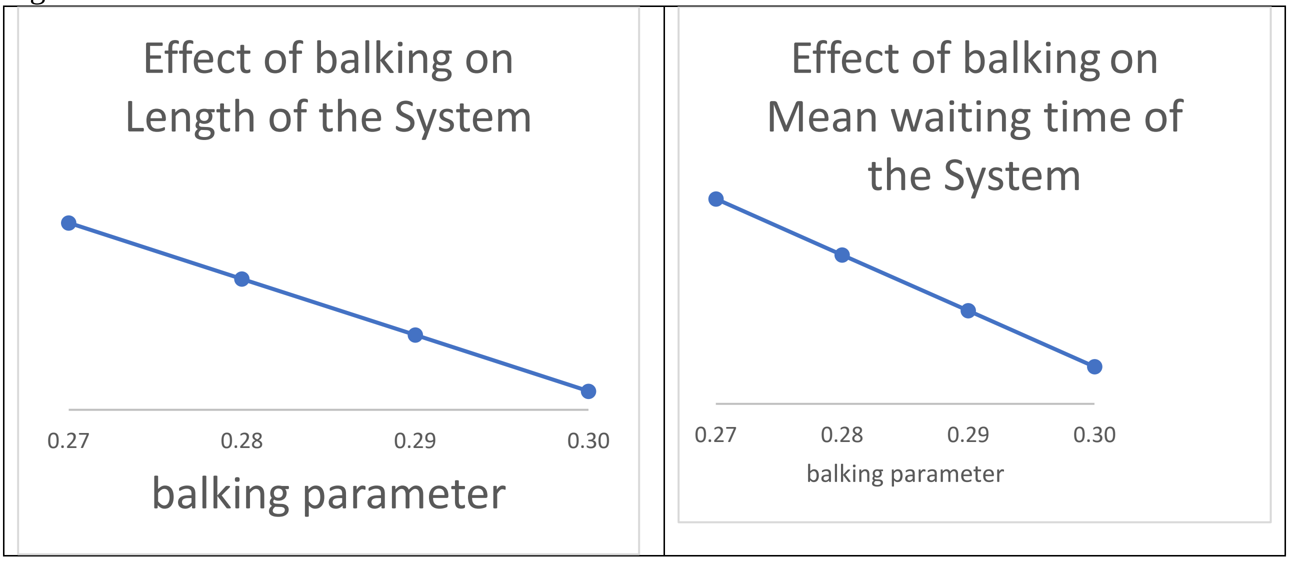
Parameter (b)	t	t_1	t_2	t_3	t_4
0.27	$L_S^{(t)}$	0.007414415	0.014660288	0.021741492	0.028661813
	$W_S^{(t)}$	0.297748544	0.590990281	0.879723586	1.1639521
0.28	$L_S^{(t)}$	0.007661562	0.015148964	0.022466209	0.029617207
	$W_S^{(t)}$	0.307711248	0.610837105	0.909370375	1.203309648
0.29	$L_S^{(t)}$	0.007908709	0.01563764	0.023190925	0.0305726
	$W_S^{(t)}$	0.317676383	0.630693387	0.939037864	1.242703004
0.3	$L_S^{(t)}$	0.008155856	0.016126316	0.023915642	0.031527994
	$W_S^{(t)}$	0.327643949	0.650559124	0.968726045	1.282132145

Figure 5:



Inference: From the above table and figure, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to increase in balking probability.

V Conclusion

The objective of this work was to detail a queueing system with two types of arrivals and state dependent service with customer's impatience. We have also demonstrated sensitivity analysis of various parameters on performance measures of the system. This work can be extended by considering a cost function and to deduce the number of servers required to optimize such total cost.

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